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ON ONE EXACT AND PERIODIC SOLUTION OF A NONLINEAR ELECTRODYNAMICS PROBLEM FOR FERROMAGNETIC AND SEGNETOELECTRIC MEDIA

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Abstract. The article considers exact and periodic solutions of nonlinear electrodynamics problems for ferromagnetic and ferroelectric media. The method of decomposition by Jacobi elliptic functions is used to obtain solutions. The solutions to the problems are found using $sn\,\xi$ — sine amplitude, $cn\,\xi$ — cosine amplitude, and $dn\,\xi$ — delta amplitude of the Jacobi function.

The foundations of the theory in electrodynamics were laid as early as in the works of L. Boltzmann, V. Volterra, D. Maxwell, and F. Voigt. Further developments in linear theory of electromagnetoelasticity belong to Yu. A. Mitropolsky, A. A. Berezovsky, Zh. L. Lyons, and their students: M. I. Razovsky, A. N. Filatov, I. K. Kurbanov, and others. The main published works in this area are qualitative studies of boundary value problems in electrodynamics and electromagnetoelasticity. In the works of I.K. Kurbanov and his students, some studies were carried out for the problems under consideration, and results were obtained on the existence and uniqueness of solutions to the problems posed. Studies of exact and periodic solutions to problems of electrodynamics and electromagnetoelasticity are devoted to works [1]–[6]. The present work is devoted to obtaining exact solutions in this area using special functions. To calculate the derivatives of the desired functions, formulas from the theory of elliptic functions are used [2].

On the plane of variables x, t we consider a system of the form

$$\begin{cases} -\dfrac{\partial H}{\partial x} = \dfrac{\partial\big(D(E)\big)}{\partial t} + J(H), \\ \dfrac{\partial E}{\partial x} = -\dfrac{\partial\big(B(E,H)\big)}{\partial t}, \end{cases} \tag{0.1}$$

with defining equations

$$D(E) = \alpha E^2, \quad J(H) = \beta\frac{\partial^3 H}{\partial x^3}, \quad B(E,H) = \mu H^2 + \widetilde{\beta}\frac{\partial^2 E}{\partial x^2}, \tag{0.2}$$

where $\alpha, \beta, \mu, \widetilde{\beta}$ are constants.

From system (1) with the help of defining equations (2), we come to the system

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$$\begin{cases} \dfrac{\partial H}{\partial x} + 2\alpha E\dfrac{\partial E}{\partial t} + \beta\dfrac{\partial^3 H}{\partial x^3} = 0, \\ \dfrac{\partial E}{\partial x} + 2\mu H\dfrac{\partial H}{\partial t} + \widetilde{\beta}\dfrac{\partial^3 E}{\partial x^2 \partial t} = 0. \end{cases} \tag{0.3}$$

In work [5], the system of equations (3) was considered with one quasi-linear equation. In this work, we will seek exact free wave solutions for the system of third-order quasi-linear equations of the form (3). To do this, using variable substitutions for the functions E and H from the variables x, t we move to the variable $\xi = k(x - ct)$, where k and c are wave numbers, i.e.

$$E(x,t) = E(\xi), \qquad H(x,t) = H(\xi). \tag{0.4}$$

From here, using (4) from system (3), we obtain the ordinary system of differential equations

$$\begin{cases} \dfrac{dH}{d\xi} - 2\alpha cE\dfrac{dE}{d\xi} + \beta k^2\dfrac{d^3 H}{d\xi^3} = 0, \\ \dfrac{dE}{d\xi} - 2\mu cH\dfrac{dH}{d\xi} - \widetilde{\beta}ck^2\dfrac{d^3 E}{d\xi^3} = 0, \end{cases} \tag{0.5}$$

where $\alpha, c, \beta, \widetilde{\beta}, \mu, k$ are constant numbers.

We will find the solution to system (5) using the sine amplitude $sn\,\xi$ of the Jacobi function of the form

$$E = a_0 + a_1\, sn^2\xi, \qquad H = b_0 + b_1\, sn^2\xi, \tag{0.6}$$

where a_0, a_1, b_0, b_1 are unknown coefficients. A similar method was used in [6] to obtain a solution for the Korteweg–de Vries equation.

Theorem 1. *Let all coefficients of the system of differential equations (5) be non-zero constants. Then the problem (1)–(3) has an exact periodic solution using* $sn\,\xi$.

Proof: Substituting (6) into the system of ordinary equations (5) and setting the coefficients at the odd powers of the function $sn\,\xi$ to zero, we obtain

$$\begin{cases} \big(1 - 4\beta k^2(m^2+1)\big)b_1 - 2\alpha c a_0 a_1 = 0, \\ -\alpha c a_1^2 + 6\beta k^2 m^2 b_1 = 0, \\ \big(1 - 4\widetilde{\beta}ck^2(m^2+1)\big)a_1 - 2\mu c b_0 b_1 = 0, \\ \mu c b_1^2 + 6\widetilde{\beta}ck^2m^2a_1 = 0. \end{cases} \tag{0.7}$$

Next, solving the system of algebraic equations (7), we determine the unknown coefficients (6) in the form

$$
\begin{cases}
a_0 = \dfrac{-\big(1-4\beta k^2(m^2+1)\big)}{2\mu c}\sqrt[3]{\dfrac{\widetilde{\beta}\alpha c}{\mu\beta}}, \quad a_1 = \sqrt[3]{\dfrac{\beta^2\widetilde{\beta}}{\mu\alpha^2c^2}}\,6k^2m^2,\\[2ex]
b_0 = \dfrac{-\big(1+4\widetilde{\beta}ck^2(m^2+1)\big)}{2\mu c}\sqrt[3]{\dfrac{\mu\beta}{\alpha c\widetilde{\beta}}}, \quad b_1 = \sqrt[3]{\dfrac{\beta\widetilde{\beta}^2}{\mu^2\alpha c}}\,6k^2m^2.
\end{cases} \tag{0.8}
$$

Substituting (8) into (6), we obtain an exact periodic solution to problem (1)–(3) using the Jacobi function $sn\,\xi$.

$$
\begin{cases}
E(x,t) = \dfrac{-\big(1-4\beta k^2(m^2+1)\big)}{2\alpha c}\sqrt[3]{\dfrac{\alpha c\widetilde{\beta}}{\mu\beta}} - 6k^2m^2\sqrt[3]{\dfrac{\beta^2\widetilde{\beta}}{\mu\alpha^2c^2}}\,sn^2\big(k(x-ct)\big),\\[2ex]
H(x,t) = \dfrac{-\big(1+4\widetilde{\beta}ck^2(m^2+1)\big)}{2\mu c}\sqrt[3]{\dfrac{\beta\mu}{\alpha c\widetilde{\beta}}} + 6k^2m^2\sqrt[3]{\dfrac{\beta\widetilde{\beta}^2}{\mu^2\alpha c}}\,sn^2\big(k(x-ct)\big).
\end{cases} \tag{0.9}
$$

Now we seek a solution to problem (1)–(3) using the cosine amplitude $cn\,\xi$ of the Jacobi function of the form

$$
E = a_0 + a_1\,cn^2\xi, \qquad H = b_0 + b_1\,cn^2\xi, \tag{0.10}
$$

where a_0, a_1, b_0, b_1 are unknown constants.

Theorem 2. *Let all coefficients of the system of differential equations (5) be non-zero constants. Then problem (1)–(3) has an exact periodic solution using $cn\,\xi$ Jacobi functions.*

Proof. Substituting (10) into the system of ordinary differential equations (5) and setting the coefficients with the same degrees $cn\,\xi$ equal to zero, we obtain a system of algebraic equations

$$
\begin{cases}
-2b_1 + 4\alpha c a_0 a_1 + 8\beta k^2(1-2m^2)b_1 = 0,\\
4\alpha c a_1^2 + 24\beta k^2m^2 b_1 = 0,\\
-a_1 + 2\mu c b_0 b_1 - 4\widetilde{\beta}ck^2(1-2m^2)a_1 = 0,\\
\mu c b_1^2 - 24\widetilde{\beta}ck^2m^2a_1 = 0.
\end{cases} \tag{0.11}
$$

From (11) we determine the unknown constants (10)

$$
\begin{cases}
a_0 = \dfrac{1-4\beta k^2(1-2m^2)}{-2\alpha c}\sqrt[3]{\dfrac{\alpha c\widetilde{\beta}}{\mu\beta}}, \quad a_1 = 6k^2m^2\sqrt[3]{\dfrac{\widetilde{\beta}\beta^2}{\alpha^2c^2\mu}},\\[2ex]
b_0 = \dfrac{-1+4\widetilde{\beta}ck^2(1-2m^2)}{-2\mu c}\sqrt[3]{\dfrac{\mu\beta}{\alpha c\widetilde{\beta}}}, \quad b_1 = -6k^2m^2\sqrt[3]{\dfrac{\widetilde{\beta}^2\beta}{\alpha c\mu^2}}.
\end{cases} \tag{0.12}
$$

From this, we obtain the following exact periodic solution of the problem (1)–(3) using the Jacobi $cn\,\xi$ function.

$$\begin{cases} E(x,t) = \dfrac{1-4\beta k^2(1-2m^2)}{-2\alpha c}\sqrt[3]{\dfrac{\alpha c\widetilde{\beta}}{\mu\beta}} + \sqrt[3]{\dfrac{\widetilde{\beta}\beta^2}{\alpha^2c^2\mu}}\,6k^2m^2\,cn^2\big(k(x-ct)\big), \\ H(x,t) = \dfrac{1+4\widetilde{\beta}ck^2(1-2m^2)}{-2\mu c}\sqrt[3]{\dfrac{\mu\beta}{\alpha c\widetilde{\beta}}} - 6k^2m^2\sqrt[3]{\dfrac{\widetilde{\beta}^2\beta}{\alpha c\mu^2}}\,cn^2\big(k(x-ct)\big). \end{cases} \tag{0.13}$$

Similarly, we will seek a solution to problems (1)–(3) using the delta amplitude $dn\,\xi$ of the Jacobi function in the form

$$E = a_0 + a_1\,dn^2\xi, \qquad H = b_0 + b_1\,dn^2\xi. \tag{0.14}$$

Proceeding as above, we find the following exact periodic solution to problem (1)–(3) using the Jacobian function $dn\,\xi$

$$\begin{cases} E(x,t) = \dfrac{1-4\beta k^2(m^2-2)}{-2\alpha c}\sqrt[3]{\dfrac{\widetilde{\beta}\alpha c}{\mu\beta}} + 6k^2\sqrt[3]{\dfrac{\widetilde{\beta}\beta^2}{\mu\alpha^2c^2}}\,dn^2\big(k(x-ct)\big), \\ H(x,t) = \dfrac{1+4\widetilde{\beta}ck^2(m^2-2)}{-2\mu c}\sqrt[3]{\dfrac{\mu\beta}{\widetilde{\beta}\alpha c}} - 6k^2\sqrt[3]{\dfrac{\widetilde{\beta}^2\beta}{\mu^2\alpha c}}\,dn^2\big(k(x-ct)\big). \end{cases} \tag{0.15}$$

In this part of the article, we consider the following problem, which is similar to the one discussed above but with stronger nonlinearity.

On the plane of variables x, t, we consider a system of the form

$$\begin{cases} -\alpha\dfrac{\partial H}{\partial x} = \dfrac{\partial\big(D(E)\big)}{\partial t} + J(E), \\ \alpha_0\dfrac{\partial E}{\partial x} = -\dfrac{\partial\big(B(H)\big)}{\partial t}, \end{cases} \tag{0.16}$$

where α, α_0 are constant numbers, with defining equations

$$\begin{gathered} D(E) = \varepsilon_0 E^3, \quad J(E) = \sigma_0 E\frac{\partial^3 E}{\partial x^2\partial t}, \\ B(H) = \mu H^2 + \widetilde{\mu}\frac{\partial^2 E}{\partial x\,\partial t}, \end{gathered} \tag{0.17}$$

where $\sigma_0, \varepsilon_0, \mu, \widetilde{\mu}$ are permanent.

From system (16) with the help of defining equations (17), we come to the system

$$\begin{gathered} -\alpha\frac{\partial H}{\partial x} = 3\varepsilon_0 E^2\frac{\partial E}{\partial t} + \sigma_0 E\frac{\partial^3 H}{\partial x^2\partial t}, \\ \alpha_0\frac{\partial E}{\partial x} = -2\mu H\frac{\partial H}{\partial t} - \widetilde{\mu}\frac{\partial^3 E}{\partial x\,\partial t^2}. \end{gathered} \tag{0.18}$$

The resulting system consists of two nonlinear equations of the third order. We can say that the first equation of the system is nonlinear, while the second is quasi-linear. We will seek exact free wave solutions for the system of nonlinear third-order equations (18). For this system, we also use variable substitution for the functions E and H from the variables x, t to the variable $\xi = k(x - ct)$, where k and c denote wave numbers and velocities, respectively, i.e.

$$E(x,t) = E(\xi), \qquad H(x,t) = H(\xi). \tag{0.19}$$

From here, using (19) from system (18), we arrive at a nonlinear ordinary system of third order of the form

$$\begin{cases} \alpha \dfrac{dH}{d\xi} - 3c\varepsilon_0 E^2 \dfrac{dE}{d\xi} - ck^2\sigma_0 E \dfrac{d^3H}{d\xi^3} = 0, \\ \alpha_0 \dfrac{dE}{d\xi} - 2\mu c H \dfrac{dH}{d\xi} - \widetilde{\mu} c^2 k^2 \dfrac{d^3E}{d\xi^3} = 0, \end{cases} \tag{0.20}$$

where $\alpha, c, \alpha_0, \widetilde{\mu}, \mu, k, \varepsilon_0, \sigma_0$ are constant numbers.

For system (20), we will seek solutions using Jacobi elliptic functions $sn\,\xi$ — sine amplitude, $cn\,\xi$ — cosine amplitude, and $dn\,\xi$ — delta amplitude.

Theorem 3. *Let all coefficients of the system of equations (20) be non-zero constants.*

Then the problem (16)–(18) has an exact periodic solution using the $sn\,\xi$ — sine amplitude of the Jacobi function only in the case $\alpha = 0$.

Proof. For the system of equations (20), based on the method of decomposition into Jacobi elliptic functions, we will seek the solution using a finite series of the form

$$E(\xi) = a_0 + a_1\, sn^2\xi, \qquad H(\xi) = b_0 + b_1\, sn^2\xi, \tag{0.21}$$

where a_0, a_1, b_0, b_1 are as yet unknown coefficients.

Thus, we substitute (21) into the ordinary nonlinear system of differential equations (20) and perform certain procedures, obtaining an overdetermined algebraic system of the following form

$$\begin{aligned} &\alpha b_1 - 3c\varepsilon_0 a_0^2 a_1 + 4ck^2\sigma_0(m^2+1)a_0a_1 = 0, \\ &-6c\varepsilon_0 a_0 a_1^2 - 4ck^2\sigma_0\big(3m^2a_0a_1 - (m^2+1)a_1^2\big) = 0, \\ &-3c\varepsilon_0 a_1^3 - 12ck^2\sigma_0 m^2 a_1^2 = 0, \\ &\alpha_0 a_1 - 2\mu c b_0 b_1 + 4\widetilde{\mu}c^2k^2(m^2+1)b_1 = 0, \\ &-2\mu c b_1^2 - 12\widetilde{\mu}c^2k^2m^2b_1 = 0. \end{aligned} \tag{0.22}$$

From there, solving the redefinite system of nonlinear algebraic equations (22), we find the unknown coefficients (21) in the form

$$
\begin{aligned}
&a_0 = \frac{1}{2}\left[\frac{4k^2\sigma_0(m^2+1)}{3\varepsilon_0} \pm \sqrt{\frac{4^2k^4\sigma_0^2(m^2+1)^2}{3^2\varepsilon_0^2} + \frac{2\widetilde{\mu}\alpha}{\mu\sigma_0}}\right],\\
&a_1 = -\frac{4k^2m^2\sigma_0}{\varepsilon_0}, \quad b_0 = \frac{1}{2\mu c}\left[\frac{2\mu\sigma_0\alpha_0}{3\widetilde{\mu}c\varepsilon_0} + 4\widetilde{\mu}c^2k^2(m^2+1)\right],\\
&b_1 = -\frac{6k^2m^2\widetilde{\mu}c}{\mu}.
\end{aligned} \tag{0.23}
$$

Solution (23) satisfies (22) under the following conditions $\mu \neq 0,\ c \neq 0,\ \widetilde{\mu} \neq 0,\ \varepsilon_0 \neq 0,\ \alpha = 0,\ \sigma_0 \neq 0$, then we obtain the solution of system (20) in the form

$$
\begin{aligned}
&E(\xi) = \frac{1}{2}\left[\frac{4k^2\sigma_0(m^2+1)}{3\varepsilon_0} \pm \sqrt{\frac{4^2k^4\sigma_0^2(m^2+1)^2}{3^2\varepsilon_0^2} + \frac{2\widetilde{\mu}\alpha}{\mu\sigma_0}}\right] - \frac{4k^2m^2\sigma_0}{\varepsilon_0}\, sn^2\xi,\\
&H(\xi) = \frac{1}{2\mu c}\left[\frac{2\mu\sigma_0\alpha_0}{3\widetilde{\mu}c\varepsilon_0} + 4\widetilde{\mu}c^2k^2(m^2+1)\right] - \frac{6k^2m^2\widetilde{\mu}c}{\mu}\, sn^2\xi
\end{aligned} \tag{0.24}
$$

at $\mu \neq 0,\ c \neq 0,\ \widetilde{\mu} \neq 0,\ \varepsilon_0 \neq 0,\ \alpha = 0,\ \sigma_0 \neq 0$.

Now we move on to the initial changes and obtain the solution to problem (16)–(18) in the form

$$
\begin{aligned}
E(x,t) = E\big(k(x-ct)\big) = \frac{1}{2}\Bigg[&\frac{4k^2\sigma_0(m^2+1)}{3\varepsilon_0}\\
&\pm\sqrt{\frac{4^2k^4\sigma_0^2(m^2+1)^2}{3^2\varepsilon_0^2} + \frac{2\widetilde{\mu}\alpha}{\mu\sigma_0}}\Bigg] - \frac{4k^2m^2\sigma_0}{\varepsilon_0}\, sn^2\big(k(x-ct)\big),\\
H(x,t) = H\big(k(x-ct)\big) = \frac{1}{2\mu c}&\left[\frac{2\mu\sigma_0\alpha_0}{3\widetilde{\mu}c\varepsilon_0} + 4\widetilde{\mu}c^2k^2(m^2+1)\right]\\
&- \frac{6k^2m^2\widetilde{\mu}c}{\mu}\, sn^2\big(k(x-ct)\big).
\end{aligned} \tag{0.25}
$$

provided that $\mu \neq 0,\ c \neq 0,\ \widetilde{\mu} \neq 0,\ \varepsilon_0 \neq 0,\ \alpha = 0,\ \sigma_0 \neq 0$.

Now we seek the solution to the substituted problem (16)–(17) using the Jacobi elliptic function decomposition method with the cosine amplitude $cn\,\xi$.

Theorem 4. *Let all coefficients of the system (20) be non-zero constants. Then the problem (16)–(17) and the system (20) have an exact periodic solution using* $cn\,\xi$ *Jacobi only in the case* $\alpha = 0$.

Proof. As above, to obtain a solution using the cosine amplitude $cn\,\xi$, we seek a solution in the form of finite series

$$
E(\xi) = a_0 + a_1\, cn^2\xi, \qquad H(\xi) = b_0 + b_1\, cn^2\xi, \tag{0.26}
$$

where a_0, a_1, b_0, b_1 are unknown constants.

And so, we substitute (26) into the ordinary nonlinear system of differential equations (20) and perform some simple transformations, equating the coefficients with the same degrees of the cosine amplitude function $cn\,\xi$ to zero, we obtain an ordinary overdetermined algebraic system of the following form

$$\begin{aligned}
&-\alpha b_1 + 3c\varepsilon_0 a_0^2 a_1 - 4ck^2\sigma_0(1-2m^2)a_0 a_1 = 0,\\
&6c\varepsilon_0 a_0 a_1^2 - ck^2\sigma_0\big(12m^2 a_0 a_1 - 4(1-2m^2)a_1^2\big) = 0,\\
&3c\varepsilon_0 a_1^3 - 12ck^2\sigma_0 m^2 a_1^2 = 0,\\
&-\alpha_0 a_1 + 2\mu c b_0 b_1 - 4\widetilde{\mu}c^2k^2(1-2m^2)b_1 = 0,\\
&2\mu c b_1^2 - 12\widetilde{\mu}c^2k^2m^2 b_1 = 0.
\end{aligned} \tag{0.27}$$

From there, we find solutions to the redefined system of nonlinear algebraic equations (27) in the form

$$\begin{aligned}
&a_0 = \frac{1}{2}\left[\frac{4k^2\sigma_0(1-2m^2)}{3\varepsilon_0} \pm \sqrt{\frac{4^2k^4\sigma_0^2(1-2m^2)^2}{3^2\varepsilon_0^2} + \frac{2\widetilde{\mu}\alpha}{\mu\sigma_0\alpha_0}}\right],\\
&a_1 = \frac{4k^2m^2\sigma_0}{\varepsilon_0}, \quad b_0 = \frac{1}{2\mu c}\left[\frac{2\mu\sigma_0\alpha_0}{3\widetilde{\mu}c\varepsilon_0} + 4\widetilde{\mu}c^2k^2(1-2m^2)\right],\\
&b_1 = \frac{6k^2m^2\widetilde{\mu}c}{\mu}.
\end{aligned} \tag{0.28}$$

Thus, using (28), we determine the solution of the system of equations (20) in the form

$$\begin{aligned}
&E(\xi) = \frac{1}{2}\left[\frac{4k^2\sigma_0(1-2m^2)}{3\varepsilon_0} \pm \sqrt{\frac{4^2k^4\sigma_0^2(1-2m^2)^2}{3^2\varepsilon_0^2} + \frac{2\widetilde{\mu}\alpha}{\mu\sigma_0\alpha_0}}\right] + \frac{4k^2m^2\sigma_0}{\varepsilon_0}\,cn^2\xi,\\
&H(\xi) = \frac{1}{2\mu c}\left[\frac{2\mu\sigma_0\alpha_0}{3\widetilde{\mu}c\varepsilon_0} + 4\widetilde{\mu}c^2k^2(1-2m^2)\right] + \frac{6k^2m^2\widetilde{\mu}c}{\mu}\,cn^2\xi.
\end{aligned} \tag{0.29}$$

Now let us move on to the initial changes and obtain the solution to problem (16)–(18) using the cosine amplitude $cn\,\xi$ in the form

$$E(x,t) = E\big(k(x-ct)\big) = \frac{1}{2}\left[\frac{4k^2\sigma_0(1-2m^2)}{3\varepsilon_0}\right.$$
$$\left.\pm\sqrt{\frac{4^2k^4\sigma_0^2(1-2m^2)^2}{3^2\varepsilon_0^2} + \frac{2\widetilde{\mu}\alpha}{\mu\sigma_0\alpha_0}}\right] + \frac{4k^2m^2\sigma_0}{\varepsilon_0}\,cn^2\big(k(x-ct)\big), \tag{0.30}$$
$$H(x,t) = H\big(k(x-ct)\big) = \frac{1}{2\mu c}\left[\frac{2\mu\sigma_0\alpha_0}{3\widetilde{\mu}c\varepsilon_0} + 4\widetilde{\mu}c^2k^2(1-2m^2)\right]$$
$$+\frac{6k^2m^2\widetilde{\mu}c}{\mu}\,cn^2\big(k(x-ct)\big).$$

provided that $\mu \neq 0,\ c \neq 0,\ \widetilde{\mu} \neq 0,\ \varepsilon_0 \neq 0,\ \alpha = 0,\ \sigma_0 \neq 0.$

Now we seek the solution to the substituted problem (16)–(18) using the Jacobi elliptic function decomposition method with the delta amplitude $dn\,\xi$.

Similarly, we find the solution to problem (16)–(18) using the delta amplitude $dn\,\xi$ of the Jacobi function.

Theorem 5. *Let all coefficients of the system (20) be non-zero constants.*

Then the problem (16)–(18) has an exact periodic solution using the delta amplitude $dn\,\xi$ Jacobi only in the case $=0$.

Proof. We seek a solution to the system of ordinary equations (20) using the delta amplitude $dn\,\xi$ Jacobi function in the form

$$E(\xi) = a_0 + a_1\,dn^2\xi, \qquad H(\xi) = b_0 + b_1\,dn^2\xi. \tag{0.31}$$

Proceeding as above, we find the following exact periodic solutions to problems (16)–(18) using the Jacobian function $dn\,\xi$. Now we substitute (31) into the ordinary system (20) and, after some transformations and calculations, arrive at the following algebraic system of the following form

$$\begin{aligned}
&-\alpha b_1 + 3c\varepsilon_0a_0^2a_1 - 4ck^2\sigma_0(m^2-2)a_0a_1 = 0,\\
&6c\varepsilon_0a_0a_1^2 - 4ck^2\sigma_0a_1\big(3a_0 + (m^2-2)a_1\big) = 0,\\
&3c\varepsilon_0a_1^3 - 12ck^2\sigma_0a_1^2 = 0,\\
&-\alpha_0a_1 + 2\mu cb_0b_1 - 4\widetilde{\mu}c^2k^2(m^2-2)b_1 = 0,\\
&2\mu cb_1^2 - 12\widetilde{\mu}c^2k^2b_1 = 0.
\end{aligned} \tag{0.32}$$

From where, we find solutions (32) in the form of

$$a_0 = \frac{1}{2}\left[\frac{4k^2\sigma_0(m^2-2)}{3\varepsilon_0} \pm \sqrt{\frac{4^2k^4\sigma_0^2(m^2-2)^2}{3^2\varepsilon_0^2} + \frac{2\widetilde{\mu}\alpha}{\mu\sigma_0\alpha_0}}\right],$$

$$a_1 = \frac{4k^2\sigma_0}{\varepsilon_0}, \quad b_0 = \frac{1}{2\mu c}\left[\frac{2\mu\sigma_0\alpha_0}{3\widetilde{\mu}c\varepsilon_0} + 4\widetilde{\mu}c^2k^2(m^2-2)\right], \tag{0.33}$$

$$b_1 = \frac{6k^2\widetilde{\mu}c}{\mu}.$$

Thus, we find the solution to the system of ordinary equations (20) in the form

$$E(\xi) = \frac{1}{2}\left[\frac{4k^2\sigma_0(m^2-2)}{3\varepsilon_0} \pm \sqrt{\frac{4^2k^4\sigma_0^2(m^2-2)^2}{3^2\varepsilon_0^2} + \frac{2\widetilde{\mu}\alpha}{\mu\sigma_0\alpha_0}}\right] + \frac{4k^2\sigma_0}{\varepsilon_0}\, dn^2\xi,$$

$$H(\xi) = \frac{1}{2\mu c}\left[\frac{2\mu\sigma_0\alpha_0}{3\widetilde{\mu}c\varepsilon_0} + 4\widetilde{\mu}c^2k^2(m^2-2)\right] + \frac{6k^2\widetilde{\mu}c}{\mu}\, dn^2\xi. \tag{0.34}$$

So, let's move on to the initial changes and find the solution to problem (16)–(18) using the delta amplitude $dn\,\xi$ Jacobian function in the form

$$E(x,t) = E\big(k(x-ct)\big) = \frac{1}{2}\left[\frac{4k^2\sigma_0(m^2-2)}{3\varepsilon_0} \pm \sqrt{\frac{4^2k^4\sigma_0^2(m^2-2)^2}{3^2\varepsilon_0^2} + \frac{2\widetilde{\mu}\alpha}{\mu\sigma_0\alpha_0}}\right] + \frac{4k^2\sigma_0}{\varepsilon_0}\, dn^2\big(k(x-ct)\big),$$

$$H(x,t) = H\big(k(x-ct)\big) = \frac{1}{2\mu c}\left[\frac{2\mu\sigma_0\alpha_0}{3\widetilde{\mu}c\varepsilon_0} + 4\widetilde{\mu}c^2k^2(m^2-2)\right] + \frac{6k^2\widetilde{\mu}c}{\mu}\, dn^2\big(k(x-ct)\big). \tag{0.35}$$

provided that $\mu \neq 0,\ c \neq 0,\ \widetilde{\mu} \neq 0,\ \varepsilon_0 \neq 0,\ \alpha = 0,\ \sigma_0 \neq 0.$

References

[1] Kudryashov N.A. *Analytical Theory of Nonlinear Differential Equations.* Moscow–Izhevsk: Institute of Computer Research, 2004. 360 p.

[2] Sikorsky Yu.S. *Elements of the Theory of Elliptic Functions: with Applications to Mechanics.* Moscow: KomKniga, 2006. 368 p.

[3] Kurbanov I. *Boundary Problems of Electrodynamics.* Kiev: Institute of Mathematics of the Academy of Sciences of the Ukrainian SSR, 1989. P. 3–23.

[4] Kurbanov I.K., Saidaliyev H.P. Exact periodic solution of the nonlinear problem of electromagnetoelasticity for a homogeneous medium // *Reports of the National Academy of Sciences of Ukraine.* 2024. Vol. 67, No. 1–2. P. 3–10.

[5] Kurbanov I.K., Saidaliyev H.P. Exact solution of Maxwell's equations in a homogeneous medium with nonlinear material equations // *Proceedings of the International Conference "Mathematics in the Constellation of Sciences", dedicated to the anniversary of Moscow*

State University Rector Academician Viktor Antonovich Sadovnichy. Moscow, April 1–2, 2024. P. 153–155.

[6] Liu S.K., Fu Z.T., Liu S.D., Zhao Q. Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations // *Physics Letters A*. 2001. Vol. 289. P. 69–74.

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