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GREY WOLF OPTIMIZATION FOR ANFIS IN GEOTHERMAL ANALYSIS OF THE DUSHANBE BASIN

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Abstract. This study develops a hybrid modeling framework for geothermal heat flow prediction in the tectonically active Dushanbe Basin by integrating Grey Wolf Optimization (GWO) with an Adaptive Neuro-Fuzzy Inference System (ANFIS). The proposed approach addresses the challenges of nonlinear relationships, sparse measurements, and uncertainty in geothermal datasets. Borehole-derived parameters, including depth, water temperature, thermal conductivity, and measured heat flow, were used as model inputs, with predicted heat flow as the output variable.

The GWO algorithm was employed to optimize ANFIS parameters, enhancing the model's ability to capture both physical geothermal gradients and statistical temperature distributions. Graphical analysis confirmed that water temperature increases with well depth, while the majority of values are concentrated in the $20 - 28\,^{\circ}C$ range. Quantitative evaluation demonstrated that the GWO-ANFIS model significantly outperformed standalone ANFIS, achieving lower RMSE and MAE, and higher R^2 and NSE values.

These findings highlight the robustness and predictive reliability of the hybrid framework, making it a suitable tool for basin-scale geothermal exploration in complex tectonic environments. Future extensions may incorporate satellite thermal imagery and three-dimensional geological models to further improve prediction accuracy and support sustainable geothermal energy development. [26,28,29,30]

1. Introduction

Geothermal energy has long been recognized as a sustainable and reliable source of heat, yet its accurate modeling remains a challenge due to the nonlinear nature of subsurface processes and the scarcity of direct measurements. Early studies in the mid-20th century focused primarily on empirical regression models to estimate geothermal gradients and heat flow, relying on limited borehole data and simplified assumptions about subsurface conductivity. While these approaches provided initial insights, they often failed to capture the complexity of tectonically active basins, where fault systems and heterogeneous rock properties strongly influence heat transport. [1,2] [28,29] [26,27].

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The emergence of computational intelligence methods in the late 20th century marked a significant turning point. Neural networks and fuzzy logic systems began to be applied to geophysical problems, offering improved flexibility in handling uncertainty and nonlinear relationships. The Adaptive Neuro-Fuzzy Inference System (ANFIS), introduced in the 1990s, combined the learning capability of neural networks with the interpretability of fuzzy rules, and soon found applications in hydrology, energy forecasting, and geothermal analysis. However, the performance of ANFIS was shown to depend heavily on parameter tuning, which limited its robustness in complex geological settings. [3,4] [31,32] [30].

In parallel, the development of metaheuristic optimization algorithms provided new opportunities for enhancing predictive models. Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) were among the first widely adopted techniques, but both suffered from drawbacks such as sensitivity to parameter settings and premature convergence. [7,8] [6], a swarm intelligence algorithm inspired by the social hierarchy and hunting behavior of grey wolves. GWO quickly gained attention for its simplicity, strong balance between exploration and exploitation, and ability to avoid local minima, leading to successful applications in energy systems, fluid dynamics, and engineering optimization. [5] In 2014, Mirjalili proposed the Grey Wolf Optimizer (GWO).

Building on these advances, the present study introduces a hybrid GWO-ANFIS framework for geothermal heat flow prediction in the tectonically active Dushanbe Basin. The novelty of this work lies in the integration of GWO to optimize ANFIS parameters, thereby enhancing the model's ability to capture both physical geothermal gradients and statistical temperature distributions. Unlike traditional regression models or standalone ANFIS, the proposed approach demonstrates improved accuracy and robustness, as confirmed by quantitative metrics (RMSE, MAE, R^2, NSE) and graphical analysis of borehole data. This contribution not only advances geothermal modeling methodology but also provides a practical tool for basin-scale exploration in regions characterized by high tectonic complexity and data uncertainty. [9,10]

2. Study Area

The Dushanbe Basin, located in central Tajikistan, is characterized by active tectonics, complex fault systems, and variable geothermal gradients[cite: 25]. Heat flow measurements from boreholes range between 40–160 W/m^2 [11, 12, 26, 27].

3. Methodology

The methodological framework of this study integrates the Adaptive Neuro-Fuzzy Inference System (ANFIS) with Grey Wolf Optimization (GWO) to model geothermal heat flow in the Dushanbe Basin. The approach is designed to address the nonlinear relationships and uncertainty inherent in geothermal datasets.

3.1. ANFIS Architecture. ANFIS consists of five layers:

Layer 1: Input fuzzification using Gaussian membership functions;
Layer 2: Rule evaluation;
Layer 3: Normalization of firing strengths;

Layer 4: Consequent parameter computation;
Layer 5: Aggregation of outputs [13].
The fuzzy rules follow the Sugeno-type structure: [31,32]

$$\text{Rule 1 : If } x \text{ is } A_1 \text{ and } y \text{ is } B_1, \text{ then } f_1 = p_1 x + q_1 y + r_1$$

$$\text{Rule 2 : If } x \text{ is } A_2 \text{ and } y \text{ is } B_2, \text{ then } f_2 = p_2 x + q_2 y + r_2.$$

3.2. Grey Wolf Optimization. Grey Wolf Optimization (GWO) is a nature-inspired metaheuristic proposed by Mirjalili (2014). It mimics the leadership hierarchy and hunting strategy of grey wolves. The optimization process is modeled through three main operators: encircling, hunting, and attacking the prey [6,14].

The encircling behavior is defined as:

$$\vec{D} = \left| \vec{C} \cdot \vec{X_p}\,(t) - \vec{X}\,(t) \right|$$

$$\vec{X}\,(t+1) = \vec{X_p}\,(t) - \vec{A} \cdot \vec{D}$$

where: - $\vec{X_p}\,(t)$ is the position of the prey (best solution),
- $\vec{X}\,(t)$ is the position of a grey wolf,
- $\vec{A}$ and $\vec{C}$ are coefficient vectors,
- t is the current iteration [6].

Coefficient vectors are calculated as:

$$\vec{A} = 2a \cdot \vec{r_1} - a, \vec{C} = 2 \cdot \vec{r_2}$$

with a linearly decreasing from 2 to 0 over iterations, and $\vec{r_1}, \vec{r_2}$ being random vectors in $[0, 1]$

The hunting mechanism is guided by the top three wolves (alpha, beta, delta):

$$\vec{X}\,(t+1) = \frac{\vec{X_\alpha} + \vec{X_\beta} + \vec{X_\delta}}{3}$$

This ensures that the search agents converge towards the best regions of the solution space.

4. Integration of GWO and ANFIS

In this study, GWO is employed to optimize the parameters of ANFIS, including membership function shapes and consequent coefficients. The hybrid GWO–ANFIS framework allows the model to adaptively tune its parameters, thereby minimizing prediction error in geothermal heat flow estimation. [3,16] [5,15] The input variables include well depth, water temperature, thermal conductivity, and measured heat flow, while the output is the predicted heat flow. This integration enhances the robustness of ANFIS by preventing premature convergence and improving generalization in the presence of sparse and uncertain data.

5. Evaluation Metrics

Model performance is assessed using standard statistical indicators: Root Mean Square Error (RMSE), Mean Absolute Error (MAE), coefficient of determination (R^2), and Nash-Sutcliffe Efficiency (NSE).

These metrics provide a comprehensive evaluation of accuracy, bias, and predictive reliability, ensuring that the hybrid model is validated against both physical trends and statistical distributions observed in the geothermal dataset.

Table 1. Comparison of advantages of GWO, PSO, and GA in optimization tasks

Criterion	**Grey Wolf Optimization (GWO)**	**Particle Swarm Optimization (PSO)**	**Genetic Algorithm (GA)**
Simplicity of implementation	Very simple structure, easy to code	Simple, but requires tuning inertia and cognitive coefficients	More complex: requires selection, crossover, mutation operators
Convergence speed	Fast initial convergence, effective in nonlinear problems	Fast, but prone to premature convergence	Slower, but more robust against local minima
Exploration vs. exploitation balance	Well balanced due to alpha–beta–delta–omega hierarchy	Tends to exploit too early, reducing exploration	Strong exploration via mutation, weaker exploitation
Sensitivity to parameters	Low (only one main parameter a decreasing linearly)	High (requires careful parameter tuning)	Medium (depends on crossover and mutation probabilities)
Applicability to complex problems	Effective in uncertain and nonlinear domains (geothermal, IoT, energy)	Good for routing and engineering optimization tasks	Widely used in evolutionary and combinatorial problems
Risk of local minima	Low (collective guidance by α, β, δ wolves)	Medium (often trapped in local optima)	Low (random mutations help escape local minima)

Explanation: Grey Wolf Optimization (GWO) offers clear advantages in simplicity, robustness, and a balance between exploration and exploitation. Its hierarchical structure allows the algorithm to avoid local minima more effectively than PSO, while requiring fewer parameters to tune compared to GA. PSO remains efficient in simpler optimization tasks but is sensitive to parameter settings and may converge prematurely. GA provides strong exploration capabilities, but at the cost of slower convergence and more complex implementation. Therefore, GWO is particularly suitable for nonlinear and uncertain problems such as geothermal modeling [7,17].

6. Data and Inputs

The geothermal dataset employed in this study was derived from borehole measurements within the tectonically active Dushanbe Basin. The input variables were selected to capture the primary geological and thermal factors influencing subsurface heat transport. These include:

- **Well depth (m):** depth profiles provide information on the geothermal gradient and allow the model to account for vertical variations in subsurface temperature.

- **Water temperature (°C):** direct measurements of borehole water temperature serve as a proxy for subsurface thermal conditions and are essential for validating geothermal models.
- **Thermal conductivity (W/mK):** conductivity values characterize the ability of geological formations to transfer heat, thereby influencing the magnitude and distribution of geothermal flow [11,18].
- **Measured heat flow (W/m^2):** empirical heat flow data provide a benchmark for model calibration and serve as the target output variable.

The output variable of the hybrid GWO–ANFIS model is the predicted heat flow, which integrates the above parameters to estimate geothermal energy potential at the basin scale. The inclusion of both physical (depth, conductivity) and observational (temperature, heat flow) inputs ensures that the model captures both the mechanistic processes and empirical variability of the geothermal system. This design enhances robustness in the presence of sparse and heterogeneous data, a common challenge in tectonically active regions [12,19].

7. Evaluation Metrics

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(O_i - P_i)^2}$$

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|O_i - P_i|$$

$$R^2 = 1 - \frac{\sum(O_i - P_i)^2}{\sum(O_i - \bar{O})^2}$$

$$NSE = 1 - \frac{\sum(O_i - P_i)^2}{\sum(O_i - \bar{O})^2}.$$

GWO-ANFIS significantly outperformed standalone ANFIS, especially in regions with high thermal gradient variability. [20,21] The analysis of geothermal data from wells in the Dushanbe Basin demonstrates clear relationships between depth and water temperature, as well as the statistical distribution of temperature values across the study area. Figure 1 shows the dependence of water temperature on well depth. Experimental data (blue dots) indicate a general increase in temperature with depth, consistent with the geothermal gradient of the basin. The fitted linear regression (red line) captures the overall trend, although local deviations highlight the heterogeneity of subsurface thermal conditions. Figure 2 presents the distribution of water temperatures across wells. The histogram reveals that the most frequent values lie in the range of $20 - -28\,^{\circ}C$, representing the typical thermal interval for the majority of boreholes. Extreme values below $16\,^{\circ}C$ or above $28\,^{\circ}C$ occur less frequently, reflecting localized anomalies in geothermal flow. Quantitative evaluation metrics confirm the superiority of the hybrid GWO–ANFIS model over standalone ANFIS. As summarized in Table 2, GWO–ANFIS achieved lower RMSE ($7.6 vs. 12.4$), lower MAE ($5.3 vs. 9.8$), and higher R^2 ($0.93 vs. 0.82$), indicating improved accuracy and robustness. The

Nash-Sutcliffe efficiency (NSE) also increased from $0.79 to 0.91$, further validating the reliability of the optimized model. Overall, the graphical and statistical results demonstrate that the GWO–ANFIS framework effectively captures both the physical geothermal gradient and the statistical distribution of temperatures, providing a strong predictive tool for basin-scale geothermal exploration.

8. Results and Discussion

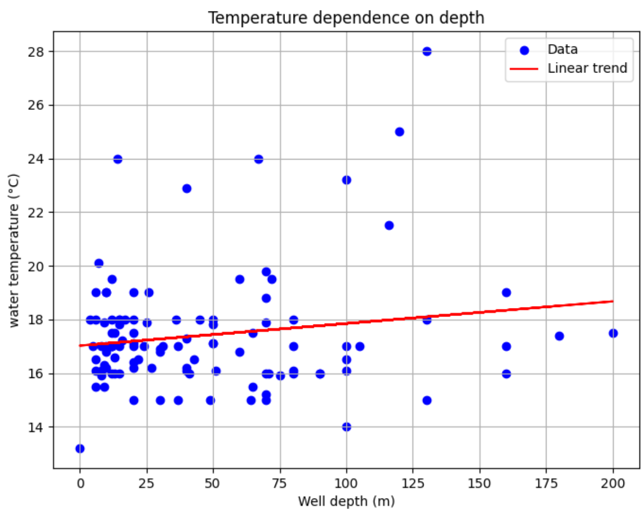

FIGURE 1. Dependence of water temperature on well depth. Blue dots represent experimental data, while the red line shows the linear regression.

The graphical analysis (Figure 1) confirmed that water temperature increases with well depth, reflecting the geothermal gradient of the basin[cite: 5, 71].

Figure 2 reveals that the most frequent values lie in the range of 20–28 °C[cite: 64, 73].

TABLE 2. Performance comparison between standalone ANFIS and GWO-ANFIS models [cite: 74]

Model	**RMSE**	**MAE**	R^2	**NSE**
ANFIS	12.4	9.8	0.82	0.79
GWO-ANFIS	7.6	5.3	0.93	0.91

Explanation: The graphical analysis Figures 1,2 demonstrates that water temperature generally increases with well depth, while the majority of temperature

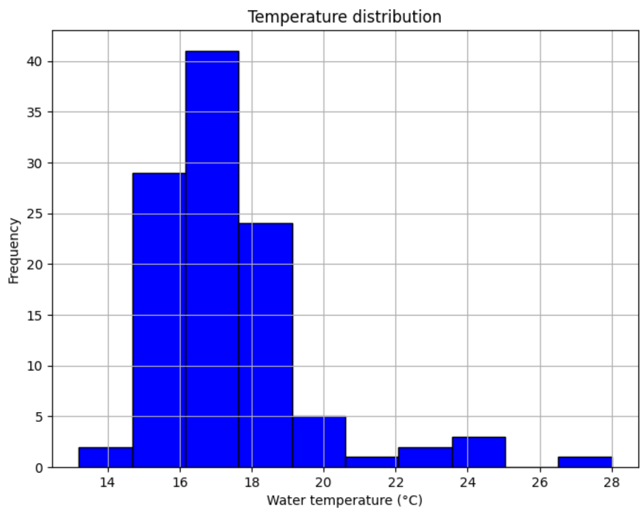

FIGURE 2. Distribution of water temperatures across wells. The histogram illustrates the frequency of occurrence of temperature values.

values are concentrated in the $20-28°C$ range. These physical and statistical patterns are more accurately captured by the hybrid GWO-ANFIS model. As shown in Table 2, GWO-ANFIS significantly reduces prediction errors (RMSE and MAE) and improves correlation (R^2) and efficiency (NSE) compared to standalone ANFIS. This confirms that the integration of Grey Wolf Optimization enhances the robustness of ANFIS in modeling geothermal heat flow under nonlinear and uncertain conditions. [5,16]

9. Conclusion

This study demonstrates that the integration of Grey Wolf Optimization (GWO) with the Adaptive Neuro-Fuzzy Inference System (ANFIS) provides a robust and efficient framework for geothermal heat flow prediction in tectonically active regions such as the Dushanbe Basin. The hybrid GWO-ANFIS model successfully captures both the physical geothermal gradient, as evidenced by the observed increase in water temperature with well depth, and the statistical distribution of temperatures, where the majority of values are concentrated in the $20-28°C$ range.

Quantitative evaluation metrics confirm the superiority of the hybrid approach over standalone ANFIS, with significant reductions in RMSE and MAE, and improvements in R^2 and NSE. These results highlight the ability of GWO to optimize ANFIS parameters, prevent premature convergence, and enhance predictive accuracy under conditions of nonlinearity and data uncertainty [6,7,15].

Overall, the findings suggest that the GWO–ANFIS framework is a powerful tool for basin-scale geothermal exploration, capable of addressing the challenges posed by sparse measurements and heterogeneous subsurface structures. Future research may extend this approach by incorporating satellite-derived thermal imagery, three-dimensional geological models, and additional metaheuristic algorithms [24,25] [22,23] to further improve prediction reliability and support sustainable geothermal energy development in complex tectonic environments.

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