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PRECISE BATCH SOLUTION OF ELECTRICAL CIRCUIT SYSTEM USING JACOBI ELLIPTICAL FUNCTIONS $SN\xi, CN\xi, DN\xi$

SAIDALIEV KH.P.

Abstract. One of the most important tasks in the theory of differential equations and systems of partial differential equations is to obtain and determine exact solutions. However, it is not always possible to find accurate solutions. In this article, we have tried to find some exact periodic solutions for the quasilinear equations of a system of electrical circuits. In order to obtain solutions in the nonlinear case, the method of elliptic decomposition of Jacobi functions was used [1]. A similar method has been used to obtain exact periodic solutions of the Korteweg-de Vries and Klein-Gordon equations in [1-6].

On surface R^2 variables (x,t) considered a system of electrical circuits of the form [3],[8]

$$\begin{cases} \frac{\partial i}{\partial x} + c(v)\frac{\partial v}{\partial t} + G(v) = 0 \\ \frac{\partial v}{\partial x} + L(i)\frac{\partial i}{\partial t} + R(i) = 0 \end{cases} \tag{1}$$

where $i(x,t)$- current, $c(v)$- capacity coefficient, $G(v)$- leakage coefficient, $v(x,t)$- voltage, $R(i)$- opposing and $L(i)$- self-induction coefficient We will consider system (1) under the following assumptions:

$$c(v) = v^2, G(v) = \frac{\partial^3 v}{\partial x^3}, R(i) = \frac{\partial i}{\partial x}, L(i) = L = const. \tag{2}$$

In system (1), instead of $c(v), G(v), R(i), L(i)$ Substituting their values accordingly (2), we get, a quasilinear system of equations of the form

$$\begin{cases} \frac{\partial i}{\partial x} + v^2\frac{\partial v}{\partial t} + \frac{\partial^3 v}{\partial x^3} = 0 \\ \frac{\partial v}{\partial x} + L\frac{\partial i}{\partial t} + \frac{\partial i}{\partial x} = 0. \end{cases} \tag{3}$$

For a system of equations (3), we will look for wave solutions using $sn\xi$- sine amplitude, $cn\xi$- cosine of amplitude and $dn\xi$- delta of amplitude of Jacobi functions [1],[4],[5].

In the system (3), by replacing variables of the form $\xi = k(x - ct)$ (where k and c constant, wave number, and wave velocity, respectively) moving on to a new change for functions

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$$i(x,t) = i(\xi), v(x,t) = v(\xi). \tag{4}$$

Thus, relatively to (4) we obtain an ordinary system of differential equations

$$\begin{cases} \frac{\partial i}{\partial \xi} + cv^2 \frac{\partial v}{\partial \xi} + k^2 \frac{\partial^3 v}{\partial \xi^3} = 0 \\ \frac{\partial v}{\partial \xi} - (cL-1)\frac{\partial i}{\partial \xi} = 0. \end{cases} \tag{5}$$

By the method of decomposition by elliptic Jacobi functions, the solution $i(\xi)$ and $v(\xi)$ can be expressed as a finite series, i.e. [1]

$$i(\xi) = \sum_{j=0}^{n} a_j sn^j \xi, v(\xi) = \sum_{j=0}^{n} b_j sn^j \xi, \tag{6}$$

where n-th term of the higher-order derivative and the nonlinear term of the equations of the system are balanced. In our case, $n = 1$ get the finite rows

$$\begin{aligned} i(\xi) =& a_0 + a_1 sn\xi, \\ v(\xi) =& b_0 + b_1 sn\xi, \end{aligned} \tag{7}$$

where a_0, a_1, b_0, and b_1 as yet unknown permanent. Thus, the solution of the system of equations (5) will be sought in the form (7).

$$\begin{gathered} \frac{d(sn\xi)}{d\xi} = cn\xi dn\xi, \frac{d(cn\xi)}{d\xi} = -sn\xi dn\xi, \frac{d(dn\xi)}{d\xi} = m^2 sn\xi cn\xi, \\ cn^2\xi + sn^2\xi = 1, dn^2\xi + m^2 sn^2\xi = 1 \text{ with module } (0 < m < 1). \end{gathered} \tag{8}$$

Using formulas (8), we get

$$\begin{aligned} \frac{\partial i}{\partial \xi} =& a_1 cn\xi dn\xi, \\ \frac{\partial v}{\partial \xi} =& b_1 cn\xi dn\xi, \\ \frac{\partial^3 v}{\partial \xi^3} =& (-(1+m^2)b_1 + 6b_1 m^2 sn^2\xi) cn\xi dn\xi. \end{aligned} \tag{9}$$

Substituting (7) and (9) in the system of equations (5), we arrive at the following system of algebraic equations

$$\begin{cases} (a_1 - cb_0^2 b_1 - (1+m^2)b_1 k^2) - 2cb_0 b_1^2 sn\xi + \\ +(6b_1 k^2 m^2 - cb_1^3) sn^2\xi = 0, \\ b_1 - (cL-1)a_1 = 0. \end{cases} \tag{10}$$

In the system (10), equating the coefficients with the same powers $sn\xi$ to zero, we have

$$\begin{cases} a_1 - cb_0^2 b_1 - (1+m^2) b_1 k^2 = 0 \\ -2cb_0 b_1^2 = 0 \\ -cb_1^3 + 6b_1 k^2 m^2 = 0, \\ b_1 - (cL-1)a_1 = 0. \end{cases} \tag{11}$$

From the system (7), we determine the unknown coefficients in the representation (11)

$$a_0 = 0, a_1 = \frac{1}{cL-1}\sqrt{\frac{6}{c}}km, b_0 = 0, b_1 = \sqrt{\frac{6}{c}}km, \tag{12}$$

which are at the $cL - 1 > 0, c > 0, k \neq 0$ and

$$m^2 + 1 = \frac{1}{k^2(cL-1)} \ or \ m^2 = \frac{1 - k^2(cL-1)}{k^2(cL-1)}, (0 < m < 1), \tag{13}$$

satisfy (11). Then we will be able to determine the exact periodic solutions of the system (5) if the condition in the form of (13)

$$\begin{aligned} i(\xi) =& \frac{\pm\sqrt{6}km}{\sqrt{c}(cL-1)} sn\xi, \\ v(\xi) =& \pm\sqrt{\frac{6}{c}}kmsn\xi, \end{aligned} \tag{14}$$

where the modulus m^2 of the elliptic function sine amplitude $(sn\xi)$ is computed by the formula

$$m^2 + 1 = \frac{1}{k^2(cL-1)} \ or \ m^2 = \frac{1 - k^2(cL-1)}{k^2(cL-1)}. \tag{15}$$

Now, moving on to the initial changes, we get the following kind of solutions

$$\begin{aligned} i(x,t) =& i(k(x-ct)) = \frac{\pm\sqrt{6}km}{\sqrt{c}(cL-1)} sn(k(x-ct)), \\ v(x,t) =& v(k(x-ct)) = \pm\sqrt{\frac{6}{c}}kmsn(k(x-ct)), \end{aligned} \tag{16}$$

So, it's proven

Theorem 1. *Let all the coefficients of the system of equations (5) be non-zero, in addition $cL - 1 \neq 0, c > 0, k \neq 0$ and the modulus of the elliptic function m^2 is calculated by formula (15). Then the system of equations (3) has an exact periodic solution in the form (16) with the modulus m^2 calculated using formula (15).*

In the same way, we will look for the solution of a system of equations (5) using $cn\xi$ Jacobi functions

$$i(\xi) = a_0 + a_1 cn\xi, v(\xi) = b_0 + b_1 cn\xi, \tag{17}$$

where a_0, a_1, b_0, and b_1 as yet unknown constants. Here, also using the formulas (8) and (17), we get

$$\begin{aligned}\frac{\partial i}{\partial \xi} =& - a_1 sn\xi dn\xi, \\ \frac{\partial v}{\partial \xi} =& - b_1 sn\xi dn\xi, \\ \frac{\partial^3 i}{\partial \xi^3} =& ((1-2m^2)b_1 + 6b_1 m^2 cn^2\xi) sn\xi dn\xi.\end{aligned} \tag{18}$$

Substituting equations in the system (5), (17) and using the formulas (18) similar to the one above, after simple transformations, we determine the coefficients a_0, a_1, b_0 and b_1 at $cL-1 \neq 0, c<0, k \neq 0$ and

$$2m^2 - 1 = \frac{1}{k^2(1-cL)} \ or \ m^2 = \frac{k^2(cL-1)-1}{2k^2(cL-1)}, (0 < m^2 < 1), \tag{19}$$

$$a_0 = 0, a_1 = \pm\sqrt{\frac{-6}{c}} \frac{1}{cL-1} km, b_0 = 0, b_1 = \pm\sqrt{\frac{-6}{c}} \frac{1}{cL-1} km. \tag{20}$$

Then the exact periodic solutions (5) are obtained in the case of $cL-1 \neq 0$, $c<0$ and $k \neq 0$ in the form of

$$\begin{aligned} i(\xi) = \pm\sqrt{\frac{-6}{c}} \frac{1}{cL-1} kmcn\xi, \\ v(\xi) = \pm\sqrt{\frac{-6}{c}} \frac{1}{cL-1} kmcn\xi, \end{aligned} \tag{21}$$

In this case, the elliptic cosine argument module ($cn\xi$) is calculated by the following formula (19)

From here, moving on to the initial changes, we get the following kind of solution

$$\begin{aligned} i(x,t) = i(k(x-ct)) = \pm\sqrt{\frac{-6}{c}} \frac{1}{cL-1} kmcn(k(x-ct)), \\ v(x,t) = v(k(x-ct)) = \pm\sqrt{\frac{-6}{c}} kmcn(k(x-ct)), \end{aligned} \tag{22}$$

when $cL-1 \neq 0, c<0$ $k \neq 0$ and module m^2 is calculated by the formula (19).

So, it's proven.

Theorem 2. *Let all the coefficients of the system of equations (5) be non-zero, in addition $cL-1 \neq 0, c<0, k \neq 0$ and an elliptic function module $cn\xi$, m^2 calculated by formula (18). Then the system of equations (3) has exact periodic solutions of the form (22) using the cosine of the amplitude - $cn\xi$ Jacobi functions.*

Now, in the same way, we find the solution of the system of equations with the help of (5) $dn\xi$ - delta of the amplitude of Jacobi functions as

$$\begin{aligned} i(\xi) &= a_0 + a_1 dn\xi, \\ v(\xi) &= b_0 + b_1 dn\xi, \end{aligned} \tag{23}$$

where a_0, a_1, b_0 and b_1 as yet unknown constants. Hence, for the definition of derivatives $i(\xi)$ and $v(\xi)$ using formulas (8), we find

$$\begin{aligned} \frac{di}{d\xi} &= -\, a_1 m^2 sn\xi cn\xi, \\ \frac{dv}{d\xi} &= -\, b_1 m^2 sn\xi cn\xi, \\ \frac{d^3 i}{d\xi^3} &= ((m^2-2)a_1 m^2 + 6a_1 m^2 dn^2\xi) sn\xi cn\xi. \end{aligned} \tag{24}$$

Substituting (23), (24) in the system of equations (5) and proceeding above by the method, we determine the unknown coefficients at $cL-1 \neq 0, c<0, k \neq 0$ and

$$m^2 - 2 = \frac{1}{k^2(cL-1)} \ or \ m^2 = \frac{1+2k^2(cL-1)}{k^2(cL-1)}, (0 < m^2 < 1) \tag{25}$$

$$a_0 = 0,\ a_1 = \pm\sqrt{\frac{-6}{c}}\frac{k}{cL-1}k,\ b_0 = 0, b_1 = \pm\sqrt{\frac{-6}{c}}k, \tag{26}$$

The obtained values of coefficients (26) satisfy our system (5) with the following relations of the form $cL-1 \neq 0, c<0, k \neq 0$ where the module m^2 elliptic functions of delta amplitude ($dn\xi$) is calculated by the following formula (25). Thus, the following exact periodic solutions of the system of equations are determined using the delta amplitude (5) ($dn\xi$) Jacobi functions

$$\begin{aligned} i(x,t) = i(k(x-ct)) &= \pm\sqrt{\frac{-6}{c}}\frac{k}{cL-1}kdn(k(x-ct)), \\ v(x,t) = v(k(x-ct)) &= \pm\sqrt{\frac{-6}{c}}kdn(k(x-ct)), \end{aligned} \tag{27}$$

moving on to the initial changes, we get

$$\begin{aligned} i(x,t) = i(k(x-ct)) &= \pm\sqrt{\frac{-6}{c}}\frac{k}{cL-1}kdn(k(x-ct)), \\ v(x,t) = v(k(x-ct)) &= \pm\sqrt{\frac{-6}{c}}kdn(k(x-ct)), \end{aligned} \tag{28}$$

at that, $cL-1 \neq 0, c<0, k \neq 0$ and m^2 - is calculated by the formula (25).

So, it's proven.

Theorem 3. *Let all the coefficients of the system of equations (5) be non-zero, in addition $cL-1 \neq 0,\ c<0,\ k \neq 0$ and an elliptic function module m^2 is calculated by formula (25). Then the system of equations (3) has an exact periodic solution of the form (28) modulo m^2 calculated formula (25).*

We will now consider systems of equations (1) with the following defining equations

$$G(v)=\frac{\partial v}{\partial x},R(i)=\frac{\partial^3 i}{\partial x^3},L(i)=i^2,c(v)=c_0=const. \tag{29}$$

By supplying (29) in a system of equations, we obtain the following quasi-linear system of equations (1)

$$\begin{aligned}&\frac{\partial i}{\partial x}+c_0\frac{\partial v}{\partial t}+\frac{\partial v}{\partial x},=0\\&\frac{\partial v}{\partial t}+i^2\frac{\partial i}{\partial i}+\frac{\partial^3 i}{\partial x^3}=0\end{aligned} \tag{30}$$

The solution of the system of equations (30) will also be searched by the method of decomposition by elliptic Jacobi functions [1].

As above, using the replacement of variables like $\xi=k(x-ct)$ moving from change x,t to variable ξ for the solution are looking for

$$i(x,t)=i(\xi),v(x,t)=v(\xi). \tag{31}$$

Then, for $i(\xi),v(\xi)$ system (30) we obtain the following system of ordinary differential equations

$$\begin{aligned}&\frac{di}{d\xi}-(cc_0-1)\frac{dv}{d\xi}=0\\&\frac{dv}{d\xi}-ci^2\frac{di}{d\xi}+k^2\frac{d^3 i}{d\xi^3}=0\end{aligned} \tag{32}$$

We will seek a solution to system (32) in the form (7).

Posing (7) and (9) in the system of equations, we obtain the following system of algebraic equations (32)

$$\begin{cases}a_1-(cc_0-1)b_1=0,\\(b_1-ca_0^2a_1-(1+m^2)a_1k^2)-2ca_0a_1^2sn\xi+\\+(6a_1k^2m^2-ca_1^3)sn^2\xi=0.\end{cases} \tag{33}$$

In the second equation of (33), equating the coefficients with the same powers of the function $sn\xi$, to zero, after simple transformations, we determine the unknown constants a_0,a_1,b_0,b_1, i.e.

$$\begin{cases}a_1-(cc_0-1)b_1=0,\\b_1-ca_0^2a_1-(1+m^2)a_1k^2=0\\-2ca_0a_1^2=0\\6a_1k^2m^2-ca_1^3=0.\end{cases} \tag{34}$$

Hence

$$a_0=0,a_1\ =\pm\sqrt{\frac{6}{c}}km,b_0=0,b_1=\pm\sqrt{\frac{6}{c}}\frac{km}{cc_0-1}. \tag{35}$$

The value found (35) satisfies when (34) at $cc_0 - 1 \neq 0, c > 0, k \neq 0$, and

$$m^2 + 1 = \frac{1}{k^2 t(cc_0 - 1)} \ \mathit{or} \ m^2 = \frac{1 - k^2(cc_0 - 1)}{k^2(cc_0 - 1)}. \tag{36}$$

In this way, we get an accurate periodic solution of the system (30) in the form of

$$\begin{aligned} i(\xi) &= \pm\sqrt{\frac{6}{c}} kmsn\xi, \\ v(\xi) &= \pm\sqrt{\frac{6}{c}} \frac{km}{cc_0 - 1} sn\xi, \end{aligned} \tag{37}$$

at that $cc_0 - 1 \neq 0, c > 0, k \neq 0$, and module m^2- elliptic functions of sine amplitude ($sn\xi$) is calculated by the formula (35).

Now, passing on to the initial changes, we obtain the exact periodic solution of the system of equations (30)

$$\begin{aligned} i(x,t) =& i(k(x - ct)) = \pm\sqrt{\frac{6}{c}} kmsn(k(x - ct)), \\ v(x,t) =& v(k(x - ct)) = \pm\sqrt{\frac{6}{c}} \frac{km}{cc_0 - 1} sn(k(x - ct)), \end{aligned} \tag{38}$$

$cc_0 - 1 \neq 0, c > 0, k \neq 0$, and module m^2- elliptic functions of sine amplitude ($sn\xi$) is calculated by the formula (35).

So, it's proven.

Theorem 4. *Let all the coefficients of the system of equations (32) be non-zero, in addition,* $cc_0 - 1 \neq 0, c > 0, k \neq 0$, *and an elliptic function module* m^2 *is calculated by formula (35). Then the system of equations (30) has exact periodic solutions of the form (38).*

In the same way, we will look for periodic solutions of the (30) system in the form of finite series (17).

Hence, supplying (17) and (18) in the ordinary system equation (32) find the following exact periodic solution, with the help of $cn\xi$ Jacobi functions, as

$$i(\xi) = \pm\sqrt{\frac{-6}{c}} kmcn\xi, v(\xi) = \pm\sqrt{\frac{-6}{c}} \frac{km}{cc_0 - 1} cn\xi, \tag{39}$$

or

$$\begin{aligned} i(k(x - ct)) &= \pm\sqrt{\frac{-6}{c}} kmcn((x - ct)), \\ v(k(x - ct)) &= \pm\sqrt{\frac{-6}{c}} \frac{km}{cc_0 - 1} cn(k(x - ct)), \end{aligned} \tag{40}$$

at $cc_0 - 1 \neq 0, c < 0, k \neq 0$ and module m^2 elliptical functions of cosine amplitude-$cn\xi$ is calculated by the formula

$$2m^2 - 1 = \frac{-1}{k^2(cc_0 - 1)} \text{ or } m^2 = \frac{k^2(cc_0 - 1) - 1}{2k^2(cc_0 - 1)}. \tag{41}$$

So, it's proven.

Theorem 5. *Let all the coefficients of the system of equations (32) be non-zero, in addition,* $cc_0 - 1 \neq 0, c < 0, k \neq 0$, *and the elliptic function module* m^2 *is calculated by the formula (40). Then the system of equations (30) has exact periodic solutions of the form (39) using* $cn\xi$ *Jacobi's functions.*

In the same way, we will look for exact periodic solutions of the (30) system in the form of finite series (23) using $dn\xi$ Jacobi functions.

So, substituting (23) and (24) in the system of equations (32) and using the above method, we obtain the following exact periodic solutions of the system (30) in the form

$$i(\xi) = \pm\sqrt{\frac{6}{-c}} k dn\xi, v(\xi) = \pm\sqrt{\frac{6}{-c}} \frac{k}{cc_0 - 1} dn\xi, \tag{42}$$

or

$$\begin{aligned} i(x,t) =& i(k(x-ct)) = \pm\sqrt{\frac{6}{-c}} k dn(k(x-ct)), \\ v(x,t) =& v(k(x-ct)) = \pm\sqrt{\frac{6}{-c}} \frac{k}{cc_0 - 1} dn(k(x-ct)), \end{aligned} \tag{43}$$

at $cc_0 - 1 \neq 0, c < 0, k \neq 0$ and elliptic delta amplitude function module $-dn\xi$ is calculated by the formula

$$m^2 - 2 = \frac{1}{k^2(cc_0 - 1)} \text{ or } m^2 = \frac{1 + 2k^2(cc_0 - 1)}{k^2(cc_0 - 1)}. \tag{44}$$

So, it's proven.

Theorem 6. *Let all the coefficients of the system of equations (32) be non-zero, in addition* $cc_0 - 1 \neq 0, c < 0, k \neq 0$, *and an elliptic function module* m^2 *is calculated by the formula (43). Then the system of equations (30) has exact periodic solutions of the form (42).*

The elliptic function expansion method is a more convenient method for obtaining exact periodic solutions to some nonlinear and quasilinear equations or a system of partial differential equations. In the second part of our article, we tried to determine the exact periodic solutions for a quasilinear system of electrical circuits (1) from the following given defining equations.

So, on the plane (x,t) we consider a system of electrical circuits of the form (1) with the following defining equations

$$c(v) = v, G(v) = \frac{\partial^3 v}{\partial x^3}, R(i) = \frac{\partial i}{\partial x}, L(i) = L = const. \tag{45}$$

Substituting (45) into the system of equations (1) we obtain the system

$$\begin{aligned} &\frac{\partial i}{\partial x} + v\frac{\partial v}{\partial t} + \frac{\partial^3 v}{\partial x^3} = 0, \\ &\frac{\partial v}{\partial x} + L\frac{\partial i}{\partial t} + \frac{\partial i}{\partial x} = 0. \end{aligned} \tag{46}$$

We will look for a wave solution to this system. To do this, we move on from variables x, t to variable ξ using variable replacement form $\xi = k(x - ct)$, for functions

$$i(x,t) = i(\xi), v(x,t) = v(\xi). \tag{47}$$

We get the following system of ordinary differential equations

$$\begin{cases} \frac{di}{\partial \xi} - cv\frac{dv}{d\xi} + k^2\frac{d^3 v}{\partial \xi^3} = 0, \\ \frac{dv}{d\xi} - cL\frac{di}{d\xi} + \frac{di}{d\xi} = 0. \end{cases} \tag{48}$$

The exact periodic solution of the system (48) will be searched for in the form of finite series by the elliptic Jacobi functions $sn\xi$, at $n = 2$ in the form of

$$\begin{aligned} i =& a_0 + a_1 sn\xi + a_2 sn^2\xi, \\ v =& b_0 + b_1 sn\xi + b_2 sn^2\xi, \end{aligned} \tag{49}$$

where $a_0, a_1, a_2, b_0, b_1, b_2$ as yet unknown constants. Using formula (8), we compute the derived functions $i(\xi)$ and $v(\xi)$

$$\begin{aligned} \frac{di}{d\xi} =& (a_1 + 2a_2 sn\xi)cn\xi dn\xi, \\ \frac{dv}{d\xi} =& (b_1 + 2b_2 sn\xi)cn\xi dn\xi, \\ \frac{d^3 v}{d\xi^3} =& (-(m^2+1)b_1 - 8(m^2+1)b_2 sn\xi + \\ &+ 6b_1 m^2 sn^2\xi + 24b_2 m^2 sn^3\xi cn\xi dn\xi. \end{aligned} \tag{50}$$

Substituting (49), (50) both in the system of equations (48) and after simple transformations, we find unknown coefficients $a_0, a_1, a_2, b_0, b_1, b_2$ in the form of

$$a_0 = 0, a_1 = 0, a_2 = \frac{12k^2m^2}{c^2L - c},$$

$$b_0 = \frac{1}{c^2L - c} - \frac{4}{c}k^2m^2(m^2+1)b_1 = 0, \quad b_2 = \frac{12k^2m^2}{c}.$$

Then we find the exact periodic solution of the system (42) at $cL - 1 \neq 0, c \neq 0$ in the form of

$$\begin{aligned} i(\xi) =& \frac{12k^2m^2}{c^2L-c}sn^2\xi, \\ v(\xi) =& \frac{1}{c^2L-c} - \frac{4}{c}k^2m^2(m^2+1) + \frac{12k^2m^2}{c}sn^2\xi, \end{aligned} \tag{51}$$

or

$$\begin{aligned} i(x,t) =& i(k(x-ct)) = \frac{12k^2m^2}{c^2L-c}sn^2(k(x-ct)), \\ v(x,t) =& v(k(x-ct)) = \frac{1}{c^2L-c} - \frac{4}{c}k^2m^2(m^2+1) + \\ &+ \frac{12k^2m^2}{c}sn^2(k(x-ct)), \end{aligned} \tag{52}$$

at

$$cL - 1 \neq 0, c \neq 0.$$

So, it's proven.

Theorem 7. *Let all the coefficients of the system of equations (48) be non-zero, in addition $cL - 1 \neq 0, c \neq 0$.*

Then the system of equations (48) has an exact periodic solution of the form (52).

Now let's look for a solution to the system (46) with the help of $cn\xi$ Jacobi functions in the form of

$$\begin{aligned} i =& a_0 + a_1 cn\xi + a_2 cn^2\xi, \\ v =& b_0 + b_1 cn\xi + b_2 cn^2\xi, \end{aligned} \tag{53}$$

where $a_0, a_1, a_2, b_0, b_1, b_2$ as yet unknown constants. Calculating derivatives $i(\xi), v(\xi)$ using (8) we find

$$\begin{aligned} \frac{di}{d\xi} =& -(a_1 + 2a_2 cn\xi) sn\xi dn\xi, \\ \frac{dv}{d\xi} =& -(b_1 + 2b_2 cn\xi) sn\xi dn\xi, \\ \frac{d^3v}{d\xi^3} =& (-(2m^2-1)b_1 - 8(2m^2-1)b_2 cn\xi + \\ &+ 6b_1 m^2 cn^2\xi + 24 b_2 m^2 cn^3\xi) sn\xi dn\xi. \end{aligned} \tag{54}$$

As above supplying (52) and (54) in the system (48) we find unknown parameters $a_0, a_1, a_2, b_0, b_1, b_2$

$$\begin{aligned} &a_0 = 0, a_1 = 0, a_2 = \frac{-12k^2m^2}{c^2L - c}, \\ &b_0 = \frac{1}{c^2L - c} + \frac{4}{c}k^2(2m^2 - 1), b_1 = 0, b_2 = \frac{-12k^2m^2}{c}. \end{aligned} \tag{55}$$

Substituting the found values of the unknown parameters (55) into (53) we obtain the following exact periodic solution of the system (46) using $cn\xi$, at $cL - 1 \neq 0, c \neq 0$,

$$\begin{aligned} i(\xi) =& -\frac{12k^2m^2}{c^2L - c}sn^2\xi, \\ v(\xi) =& \frac{1}{c(cL - 1)} + \frac{4k^2(2m^2 + 1)}{c} - \frac{12k^2m^2}{c}cn^2\xi, \end{aligned} \tag{56}$$

or

$$\begin{aligned} i(x,t) =& i(k(x - ct)) = -\frac{12k^2m^2}{c^2L - c}sn^2(k(x - ct)), \\ v(x,t) =& v(k(x - ct)) = \frac{1}{c(cL - 1)} + \frac{4k^2(2m^2 + 1)}{c} - \\ & - \frac{12k^2m^2}{c}cn^2(k(x - ct)), \end{aligned} \tag{57}$$

at $cL - 1 \neq 0, c \neq 0$.

So, it's proven.

Theorem 8. *Let all the coefficients of the system of equations (48) be non-zero, in addition, $cL - 1 \neq 0, c \neq 0$.*

Then the system of equations (46) has an exact periodic solution with respect to $cn\xi$ Jacobi functions of the form (56) or (57).

In a similar way, we will look for the solution of the system of equations by the method of decomposition by elliptic Jacobi functions with respect to $dn\xi$ i.e.

$$\begin{aligned} i =& a_0 + a_1 dn\xi + a_2 dn^2\xi, \\ v =& b_0 + b_1 dn\xi + b_2 dn^2\xi, \end{aligned} \tag{58}$$

where $a_0, a_1, a_2, b_0, b_1, b_2$ as yet unknown constants, using (8) we get

$$\begin{aligned} \frac{di}{d\xi} =& -(a_1m^2 + 2a_2m^2dn\xi)sn\xi cn\xi, \\ \frac{dv}{d\xi} =& -(b_1m^2 + 2b_2m^2dn\xi)sn\xi cn\xi, \\ \frac{d^3v}{d\xi^3}&(b_1m^2(m^2 - 2) + 8m^2(m^2 - 2)dn\xi + \\ & + 6b_1m^2dn^2\xi + 24b_2m^2dn^3\xi)cn\xi sn\xi. \end{aligned} \tag{59}$$

Substituting (58) and (59) into the system of equations (48) we obtain

$$\begin{aligned}
&-a_1m^2-2a_2m^2dn\xi+cb_0b_1m^2+m^2(2cb_0b_2+cb_1^2)dn\xi+\\
&+3cb_1b_2m^2dn^2\xi+2cb_2^2m^2dn^2\xi+b_1k^2m^2(m^2-2)+\\
&+8k^2b_2m^2(m^2-2)dn\xi+6k^2b_1m^2dn^3\xi+24k^2b_2m^2dn^3\xi=0,\\
&-b_1m^2-2b_2m^2dn\xi+cLa_1m^2+2cLa_2m^2dn\xi-a_1m^2-2a_2m^2dn\xi=0.
\end{aligned} \tag{60}$$

Hence, equating the coefficients with the same powers, the function $dn\xi$ to zero we get the following algebraic system.

$$\begin{aligned}
&-a_1m^2+cb_0b_1m^2+b_1k^2m^2(m^2-2)=0,\\
&-2a_2m^2+c(2b_0b_2+b_1^2)m^2+8k^2b_2m^2(m^2-2)=0,\\
&3cb_1b_2m^2+6k^2b_1m^2=0,\\
&2cb_2^2m^2+24k^2b_2m^2=0,\\
&-b_1m^2+cLa_1m^2-a_1m^2=0,\\
&-2b_2m^2+2cLa_2m^2-2a_2m^2=0.
\end{aligned} \tag{61}$$

From this system, at $cL-1\neq 0, c\neq 0$, find

$$\begin{aligned}
&a_0=0, a_1=0, a_2=-\frac{12k^2}{c(cL-1)},\\
&b_0=\frac{1}{c^2L-c}-\frac{4}{c}k^2(m^2-2), b_1=0, b_2=-\frac{12k^2}{c}.
\end{aligned} \tag{62}$$

From this it follows that the exact periodic solution of the system of equations (46) can be determined in the case of $cL-1\neq 0, c\neq 0$. Thus, substituting the found value (62) in (47), we find the solution of the system, equation (48) in the form

$$\begin{aligned}
&i=-\frac{12k^2}{c^2L-c}dn^2\xi,\\
&v=\frac{1}{c^2L-c}-\frac{4}{c}k^2(m^2-2)-\frac{12k^2}{c}dn^2\xi.
\end{aligned} \tag{63}$$

or

$$\begin{aligned}
&i(x,t)=i(k(x-ct))=-\frac{12k^2}{c^2L-c}dn^2(k(x-ct)),\\
&v(x,t)=v(k(x-ct))=\frac{1}{c^2L-c}-\frac{4}{c}k^2(m^2-2)-\frac{12k^2}{c}dn^2(k(x-ct)).
\end{aligned} \tag{64}$$

at

$$cL-1\neq 0, c\neq 0.$$

So, it's proven.

Theorem 9. *Let all the coefficients of the system of equations (48) be non-zero, in addition $cL - 1 \neq 0, c \neq 0$.*

Then the system of equations (46) has an exact periodic solution of the form (64).

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SAIDALIEV KH.P.: BOKHTAR STATE UNIVERSITY
Email address: homid-1978@mail.ru