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INVESTIGATION OF THE SOLUTION OF THE INTEGRO-DIFFERENTIAL PROBLEM OF PLANT PROTECTION BY NUMERICAL METHOD AND AN ALGORITHM FOR DETERMINING UNKNOWN PARAMETERS

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Abstract. The article describes the study of solving the integro-differential problem of plant protection from pests using a numerical method and an algorithm for determining unknown parameters. In the work, a mathematical model of a biological system was developed, including a description of the interaction of biological species with arbitrary trophic functions. The article discusses the issues of difference approximation of the plant protection problem and the convergence of the solution of the corresponding difference approximating problem to the solution of the initial differential plant protection problem. In the numerical solution of the problem of plant protection, many parameters are not always determined using field experiments. Based on this, an appropriate algorithm has been developed to determine unknown parameters, which is based on the least squares method.

1. Introduction

In the process of mathematical modeling of most problems in physics, biology, mechanics and other fields of science, differential equations, ordinary or partial derivatives are used. The use of integro-differential equations in mathematical models of the problem of plant protection is one of the methods of analysis and forecasting of biological systems. The difference equations were applied in [May R, 1974], in which the term "chaos" was used in the model description of biological populations using a logistic equation. In [Frisman, Kulakov, 2023], the bifurcations of the systems of two migrationally related populations described by the Bazykin model or the Riker model are considered. The work [V.A. Chetverbotsky, A. N. Chetverbotsky, 2020] examines mathematical models of the dynamics of the soil–plant system, which include agricultural plants, elements of mineral nutrition of plants of their mobile and stationary forms and microorganisms of the rhizosphere. The analytical approach is used to study the stability of stationary solutions in the case of local interaction and calculations based on the direct method to account for diffusion and advective processes [Ha, Cibulin, 2021], which considers a one-dimensional spatial problem for a heterogeneous resource and three

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types of taxis (victims per resource from predator and predator to victim). Mathematical biology is based on the mathematical theory of population dynamics, which gives ideas about the dynamics of the number of insect and plant species and their interaction in the form of mathematical formulas using systems of differential, integro-differential and difference equations [Murray J., 1983]. The work [Yunushi, Raimzoda 2016] is devoted to solving a nonlinear problem related to partial differential equations satisfying initial and boundary conditions. In [Raimzoda 2021], a mathematical model of population waves in nonlinear systems is investigated, taking into account time-age and spatial distributions. The numerical solution of nonlinear differential equations under given boundary conditions can be constructed using variational or grid methods. In solving systems of differential equations by numerical method, an error occurs, which may depend on the chosen method or the given initial conditions.

This article continues the cycle of research works [Odinaev 2019, Odinaev, Gaforov 2021]. In these works, a model biosystem of the plant protection problem of three trophic levels "cotton – harmful insects – beneficial insects" was formulated and studied, taking into account the age structure and with arbitrary trophic functions representing a system of differential equations. The necessary and sufficient conditions for the existence of a solution to the preparatory problem in stationary and non-stationary cases are found. The mathematical formulation of the plant protection problem can be described as follows:

The notation is introduced $N_1^{\tau} = \frac{1}{\tau}\int_0^{\tau} N_1(t)dt$- the average biomass of a plant (or average yield) at a given time $\tau, \widetilde{N}_i(t) = \frac{1}{\tau}\int_0^{\tau} \widetilde{N}_i(t)dt, i = 2, 3 where \tau > 0$- the total numbers of harmful and beneficial insects. You need to find the numbers N_2^p, N_3^p, so that the conditions are met $\frac{1}{\tau}\int_0^{\tau} \widetilde{N}_2(t)dt \leq N_2^p$ and $\frac{1}{\tau}\int_0^{\tau} \widetilde{N}_3(t)dt \geq N_3^p$, for which, the condition is guaranteed to be fulfilled

$$\frac{1}{\tau}\int_0^{\tau} N_1(t)dt \geq N_1^p, N_1^p \in \left[N_1^{min}, N_1^{max}\right],$$

where N_1^p- the set value of the planned harvest of cotton biomass. N_2^p- the threshold of harmfulness of pests, N_3^p-the level of effectiveness of beneficial insects (entomophages). Since in these studies all mathematical models are described using a system of differential equations, ordinary or partial derivatives, and solving such problems is often analytically impossible, in such cases numerical methods are used to find solutions to problems. Therefore, this work is devoted to the study of solving the integro-differential problem of plant protection by a numerical method and an algorithm for determining unknown parameters.

2. Numerical method for solving the integro-differential problem of plant protection

Let's consider a mathematical model of a biosystem of the type "harmful insects - beneficial insects" with arbitrary trophic functions. Suppose that the state of the model biosystem, taking into account the age structure and with arbitrary trophic functions, is described using the following system of differential equations [Odinaev 2019, Odinaev, Gaforov 2021]:

$$\begin{cases} \frac{dN_0}{dt} = Q - \alpha_0 N_0 N_1, \\ \frac{dN_1}{dt} = k_0 V_0\left(N_0\right) N_1 - V_1\left(N_1\right) \widetilde{N}_2 - m_1 N_1, \\ \frac{\partial N_2}{\partial t} + \frac{\partial N_2}{\partial a} = k_1 V_1\left(N_1\right) N_2 - V_2\left(N_2\right) \widetilde{N}_3 - m_2 N_2, \\ \frac{\partial N_3}{\partial t} + \frac{\partial N_3}{\partial a} = k_2 V_2\left(N_2\right) N_3 - \varepsilon N_3^2 - m_3 N_3, \\ N_i\big|_{t=0} = N_i^0, N_i(0,t) = \int_{\alpha_i}^{\beta_i} B_i(\xi) N_i(\xi,t) d\xi, i = 2,3, \\ \widetilde{N} = \int_{\alpha_i}^{\alpha_i} N_i(a,t) da, i = 2,3, \end{cases} \tag{1}$$

where Q is the rate of receipt of an external resource, $\widetilde{N}_i = \widetilde{N}_i(t)$- total numbers of harmful and beneficial insects $i = 2,3$. $N_0(t)$- the mass of the external resource at time t (fertilizer, or water used for irrigation, or solar energy), $N_1 = N_1(t)$- biomass of agricultural crops at a time t, $N_i = N_i(a,t)$- the number of harmful $(i = 2)$ and beneficial $(i = 3)$ insects of age a, at time t. $B_2(a), B_3(a)$- fertility rates of harmful and beneficial insects, V_i- an arbitrary trophic function with properties: $V_i(\cdot) \leq 0, \frac{dV_i}{dN} > 0, \frac{d^2V_i}{dN^2} \leq 0, i = 1,2, k_i, m_i, \alpha_i, \beta_i, \alpha_i, \alpha_i, \varepsilon$-specified non-negative constants, $\varepsilon > 0$.

Let's introduce the replacement of variables $t = a+\tau, M_i(a,\tau) = N_i(a, a+\tau), i = 2,3$; and we will rewrite task (1) in the form

$$\begin{cases} \frac{dN_0}{dt} = Q - \alpha_0 N_0 N_1, \\ \frac{dN_1}{dt} = k_0 V_0(N_0) N_1 - V_1(N_1) \widetilde{N}_2 - m_1 N_1, \\ \frac{\partial M_2}{\partial a} = k_1 V_1(N_1) M_2 - V_2(M_2) \widetilde{M}_3 - m_2 M_2, \\ \frac{\partial M_3}{\partial a} = k_2 V_2(M_2) M_3 - \varepsilon M_3^2 - m_3 M_3. \end{cases} \tag{2}$$

It is easy to see that the solution of the latter problem is presented in the following proportions:

$$N_0(t) = N_0\left(0\right) exp(-\alpha_0 \int_0^t N_1(\tau) d\tau) + Q \int_0^t exp(-\alpha_0 \int_\tau^t N_1\left(\xi\right) d\xi) d\tau.$$

$$N_1(t) = N_1(0) exp\bigg(k_0 \int_0^t V_0(N_0(\xi)) d\xi - \int_0^t \frac{V_1(N_1(\xi)) N_2(\xi) d\xi}{N_1(\xi)} - m_1 t\bigg),$$

$$M_2(a,\tau) = M_2(0,\tau) exp\Bigg(k_1 \int_0^a V_1(N_1(\xi+\tau)) d\xi - \int_0^a \frac{V_2(M_2)\widetilde{M}_3 d\xi}{M_2(\xi,\tau)} - m_2 a\Bigg) \tag{3}$$

$$M_3(a,\tau) = \frac{M_3(0,\tau) exp\big(k_2 \int_0^a V_2(M_2(\xi,\xi+\tau)) d\xi - m_3 a\big)}{1 + \varepsilon M_3(0,\tau) \int_0^a e^{k_2 \int_0^\xi V_2 d\xi - m_3 \xi} d\xi},$$

and therefore, to determine the unknown functions, we obtain the following system of integral equations:

$$N_0(t) = N_0(0)\exp\left(-\alpha_0\int_0^t N_1(\xi)\,d\xi\right) + \int_0^t Q(\xi)\exp\left(-\alpha_0\int_\xi^t N_1(s)\,ds\right)d\xi,$$

$$N_1(t) = N_1(0)\exp\left(k_0\int_0^t V_0(N_0(\xi))\,d\xi - \int_0^t \frac{V_1(N_1(\xi))\int_{a_3}^{\pi} N_2(s,t+\xi-a)\,ds}{N_1(\xi)}\,d\xi - m_1 t\right),$$

$$N_2(a,t) = \left(\int_{a_2}^{\beta_2} B_2(\xi)N_2(\xi,t)\,d\xi\right)\times\exp\left(k_1\int_0^a V_1(N_1(\xi+t-a))\,d\xi - m_2 a\right.$$

$$\left.-\int_0^a \frac{V_2(M_2(\xi,\xi+t-a))\int_{a_3}^{\pi} N_3(\eta,t-a+\xi)\,d\eta}{M_2(\xi,t+a-\xi)}\,d\xi\right),$$

$$M_3(a,\tau) = \frac{\int_{a_3}^{\beta_3} B_3(\xi)N_3(\xi,t)\,d\xi\times\exp\left(k_2\int_0^a V_2(N_2(\xi,T-\tau+\xi))\,d\xi - m_3 a\right)}{1+\epsilon\int_{a_3}^{\beta_3} B_3(\xi)N_3(\xi,t)\,d\xi\times\int_0^a \exp\left(k_2\int_0^\xi V_2(M_2(\eta,t-a+\eta))\,d\eta - m_3\xi\right)d\xi}. \quad (4)$$

In formulas (4), trophic functions $V_i(\cdot)$ are sufficiently derived. Let us consider formula (4) for the case when trophic functions are determined by Vito Volterra's law [Volterra 1976, Svirezhev 1978], i.e.

$$V_i(N) = \alpha_i N, i = 0,1,2. \quad (5)$$

Then, taking into account (5), the second, third and fourth equations of formula (4) take the following form:

$$N_0(t) = N_0\,(0)\,exp(-\alpha_0\int_0^t N_1(\tau)d\tau) + Q\int_0^t exp(-\alpha_0\int_\tau^t N_1\,(\xi)d\xi)d\tau,$$

$$N_1(t) = N_1(0)\exp\left(k_0\alpha_0\int_0^t N_0(\xi)d\xi - \int_0^t\int_{\bar{a}_2}^{\bar{\bar{a}}_2} N_2\,(\eta,t-a+\xi)\,d\eta d\xi - m_1 t\right)$$

$$N_2(a,t) = \int B_2(\xi)N_2(\xi,t)d\xi\exp\left(k_1\alpha_1\int_0^a N_1(\xi+t-a)d\xi - m_2 a -\right.$$

$$\left.\alpha_2\int_0^a M_2(\xi,\xi+t-a)\int_{\bar{a}_3}^{\bar{\bar{a}}_3} N_3(\eta,t-a+\xi)d\eta d\xi\right)$$

$$M_3(a,\tau) = \frac{\int_{\alpha_3}^{\beta_3} B_3(\xi)N_3(\xi,t)d\xi exp\big(k_2\alpha_2\int_0^a N_2(\xi,t-\tau+\xi)d\xi - m_3 a\big)}{1+\varepsilon\int_{\alpha_3}^{\beta_3} B_3(\xi)N_3(\xi,t)d\xi\int_0^a exp\Big(k_2\int_0^\xi N_2(\eta,t-a+\eta)d\eta - m_3\xi\Big)d\xi}. \quad (6)$$

Let's introduce grids and notation: $h = (h_1, h_2) > 0, G_1^{n_1} = \left\{a : a = ih_1, i = \overline{0,k_1}\right\}$, $G_2^{h_2} = \left\{t : t = jh_2, j = \overline{0,k_2}\right\}, G^h = G_1^h \cup G_2^h$,

$$Y_m^h = N_m(a,t), m = 0,1,2,3; (a,t) \in G^h,$$

$$\begin{cases} Y_0^h = Y_0(0)exp(-\alpha_0 \sum_{t \in G_2^{h_1}} Y_1^{h_1}(t)h_1) + \sum_{t \in G_2^{h_2}} Q(t)exp(-\alpha_0 \sum Y_1^{h_1}(t)h_1)h_1 \\ Y_1^h = Y_1(0)exp\left(-k_0\alpha_0 \sum_{t \in G_2^{h_1}} Y_0^{h_1}(t)h_1)\right) - \alpha_1 \left(\sum_{t \in G_{2t}^{h_2}} \left(\sum Y_2(a, t-a+\xi)h_1\right) h_2 - m_1 t\right) \\ Y_2^h(a,t) = \sum B_2(a)Y_2(a,t)h_2 exp((k_1\alpha_1 \sum Y_1(t+\xi-a)\, h_2) - m_2 a - \alpha_2 \sum N_2(\xi, \xi+t-a) \sum Y_3(\eta, t-a+\xi))^2 \\ Y_3^h(a,t) = \frac{\sum \xi B_3(\xi) Y_3(\xi,t) h_2 exp(k_2\alpha_2 \sum Y_2(\xi, t-a+\xi)h_2 - m_3 a)}{1+\varepsilon \sum \xi B_3(\xi) N_3(\xi,t) h_2 \sum \xi exp(k_2 \sum N_2(\eta, t-a+\eta)h_2 - m_3\xi)}. \end{cases} \tag{7}$$

The convergence of the solution of the difference problem (7) to the solution of the differential problem (1) is established in the usual way.

When conducting numerical experiments using formulas (7), we will use explicit algorithms of formulas (7). This means that in the right-hand parts (7), all calculations will depend on previous points in time, at which the values of the desired functions are already known.

Therefore, we will have explicit formulas for the definition $Y_i^h(t)$.

In formulas (7), many parameters of the problem are not always determined using natural experiments. Based on this, we will indicate below the method of their determination, which is based on the least squares method.

3. Algorithm for determining unknown parameters in the problem of plant protection

Consider a model ecosystem taking into account time-age distributions:

$$\begin{cases} D_{ta}N = F(N,a,t), 0 < a < a_k, 0 < t \le t_k \\ N(a,0) = N_0(a), 0 \le a \le a_{max} \\ N(a,0) = \int_0^{a_{max}} \int B(N(\xi,t),\xi,t)d\xi, 0 \le t \le t_k. \end{cases} \tag{8}$$

where $D_{ta} = \frac{\partial}{\partial t} + \frac{\partial}{\partial a}, F(\cdot), B(\cdot)$- accordingly, the vector functions of mortality and fertility, $N = (N_1, ..., N_m), N_i = N_i(a,t)$ the number of the i-th species of age a at the moment of time $t, i = \overline{1,m}.$ Let $F_i(\cdot) = b_i N_i + \sum_{j=1}^m A_{ij} N_i N_j$, where $b_i A_{ij}$- biological parameters of the population and the results of monitoring the state of the number of ecosystem species at a time t_j:

$N_{ijk} = N_i(a_k, t_j) + \xi_{ijk}, i = \overline{1,m}.j = \overline{1,n_0}, k = \overline{1,k_0}$, where ξ_{ijk}-observation errors satisfying the conditions:

$$M\left[\xi_{ijk}\right] = 0, M\left[\overline{\xi_{jk}, \xi_{jk}}\right] = \wedge(a_k, t_j)$$

Let's also assume that the coefficients b_i and functions of fertility B_i are given. It is required to define the matrix of interaction of A with the elements A_{ij}. To solve this problem, consider the functionality

$$I(A) = \sum_{k,j=1}^{k_0,n_0} P_{jk} \sum_{j=1}^{m} \left[N_{ijk} - N_i(a_k, t_j, A)\right]^2, \tag{9}$$

where P_{jk}- weight coefficients, moreover $\sum k, jP_{jk} = 1, P_{jk} \ge 0, N_i(a_k, t_j, A)-$ solving the problem (8) at the point $(a_k, t_j), k = \overline{1,k_0}, j = \overline{1,n_0}$. The minimization of the functional (9) will be carried out by the gradient descent method, slightly modified in order to obtain a sufficiently accurate solution. Here is an algorithm for solving the problem.

1) lets A^S- it is known, i.e. the s-th approximation is given.

2) We solve the problem (1).

3) Calculate $I(A^S)$.

4) Solving the sensitivity problem

$$\begin{cases} D_{ta}\left(\dfrac{\partial N_i}{\partial A_{\alpha\beta}}\right) = \begin{cases} b_i \frac{\partial N_i}{\partial A_{\alpha\beta}} + \sum_j A_{ij}\left(\frac{\partial N_i}{\partial A_{\alpha\beta}} N_j + \frac{\partial N_j}{\partial A_{\alpha\beta}} N_j\right), & a \neq i \\ b_i \frac{\partial N_i}{\partial A_{\alpha\beta}} + \sum_j A_{ij}\left(\frac{\partial N_i}{\partial A_{\alpha\beta}} N_j + \frac{\partial N_j}{\partial A_{\alpha\beta}} N_j\right) + N_\alpha N_\beta, & a = i \end{cases} \\ \left.\dfrac{\partial N_i}{\partial A_{\alpha\beta}}\right|_{t=0} = 0, \quad \left.\dfrac{\partial N_i}{\partial A_{\alpha\beta}}\right|_{a=0} = \displaystyle\int_0^{a_{\max}} \dfrac{\partial B_i}{\partial A_{\alpha\beta}}\, d\xi \end{cases}$$

5) We find the gradient vector of the functional

$$\nabla^S_{\alpha\beta} = -2\sum_{k,j} P_{jk} \sum_{j=1} \left[N_{ijk} - N_1\left(a_k, t_j, A^S\right)\right] \left.\frac{\partial N_i}{\partial A_{\alpha\beta}}\right|_{\alpha_k, t_j, A^S}$$

6) Calculate the value

$$\|\Delta S^S\| = \sqrt{\sum \alpha, \beta \left(\nabla^S_{\alpha\beta} S^2\right) dS} = \|\Delta S^S\|/L$$

where L is the scale factor, $L = const > 0$.

7) Using a random number sensor, we generate l_0 random vectors uniformly distributed on a hypersphere of radius 1 in space R^n.

8) Calculate l_0 the directions of random descent using the formula

$$\Delta^{S,1}_{\alpha,\beta} = \Delta^S_{\alpha,\beta} + d^S \eta^l_{\alpha,\beta}, l = \overline{1,..,l_0},$$

where $\eta^l_{\alpha,\beta}$ - the components of a random vector obtained in the previous paragraph.

9) Calculate A^{S+1}:

$$A^{S+1}_{\alpha,\beta} = A^S_{\alpha,\beta} - \rho^S_e \Delta^{S,l}_{\alpha,\beta}$$

where ρ^S_e It is determined from solving the following minimization problem:

$$\rho^S_e = \arg\min_{\rho\in[0,1]} \left\| \left(A^{S,1}_{\alpha,\beta} - \rho\, \Delta^{S,l}_{\alpha,\beta}\right) - A^*_{\alpha,\beta} \right\|$$

10) If no value is found ρ^S_e, not for any random direction $l = \overline{1,..,l_0}$, then we reduce the constant L in paragraph 6 and repeat the calculations in paragraphs 7-9.

11) Otherwise, we define the value $I(A^{s+1}), A^{s+1} = A^{s+1,l}$.

The iterative process continues until the required accuracy is reached, or until the iteration limit is used up. The stability and convergence of the above algorithm is established in the usual way. Note that the proposed algorithm is easily applied in cases of time–age and spatial models.

Remark. From numerical experiments, it became known that at the point of the global minimum, the value of the matrix may be ambiguous, i.e. the functional may have several local minima. For the successful practical application of the algorithm, guided by the biological meaning of the elements of the matrix of interaction of the agrocenosis, it is necessary to choose the initial approximation as close as possible to the point of the global minimum.

Example. Consider the "spider mite-tick-eating thrips" system.

$$\begin{cases} D_{ta}N_1 = b_1 N_1 - \alpha N_1 N_2 - \varepsilon_1 N_1^2 \\ D_{ta}N_2 = -b_2 N_2 + \kappa\alpha N_1 N_2 - \varepsilon_2 N_1^2 \\ N_1(a,0) = N_i^0(a), \quad N_i(0,t) = \int_0^{a_{\max}} B_i(a) N_i(a,t)\, da \end{cases} \tag{10}$$

where N_1- is the number of spider mites, N_2– the number of tick-eating thrips, $b_1, b_2, \alpha, k, \varepsilon_i$- their biological parameters, N_i^0-initial population, $B_i(a)$ – the birth rates of spider mites and tick-eating thrips, respectively. In the model biosystem (10), the coefficients b_1, b_2 the coefficients are set, and α, k, ε_i they are determined according to the above algorithm. The results of the calculation of this example were carried out using a computer program created in the Mathlab environment.

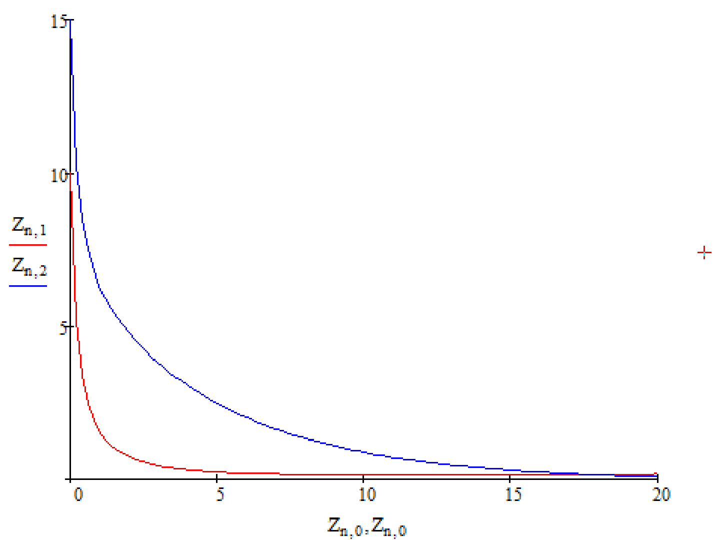

Figure 1. The results of computational experiments for the system "spider mite - tick-eating thrips (Zn,1, Zn,2, respectively)".

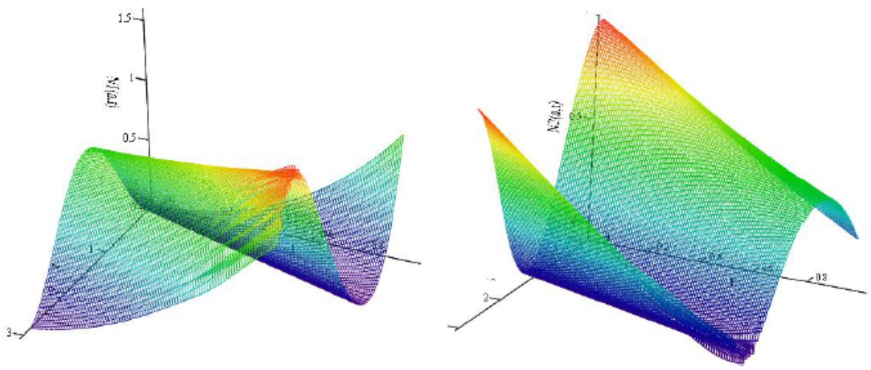

Figure 2. Phase portrait of the system "spider mite - tick-eating thrips (N1(a,t), N2(a,t)-respectively)".

4. Conclusion

In this article, numerical methods for solving the integro-differential problem of plant protection were considered. The described algorithm makes it possible to effectively determine unknown parameters in the problem of plant protection. Thus, this study has great practical significance. It follows from the results obtained that the developed methodology can be used in the design of measures to protect plants from pests of agrocenoses and to solve the problem of forecasting, planning, conducting field experiments for specific populations, biological communities and ecological systems.

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