Journal of Applied Data Analysis and Modern Stochastic Modelling
Vol. 2 No. 1 (June, 2025)

MATHEMATICAL MODELING OF THE IMPACT OF CLIMATE CHANGE ON THE GROWTH OF PESTS AND USEFUL INSECTS

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Аннотация. The presented article focuses on the mathematical and computational modeling of the impact of climate change on the growth of harmful and beneficial insect populations. In this research, a differential model for insect population growth has been developed, taking into account key climate parameters such as temperature, humidity, precipitation, and solar radiation. This model, described through a system of differential equations, allows us to study the dynamics of harmful and beneficial insect populations under changing climatic conditions. Using simplified systems of equations and analytical methods, the model has been analyzed in a simplified form. To streamline the equations, key climate parameters such as soil moisture, temperature, and solar radiation were considered constants, facilitating calculations and improving understanding of the general impact of climate on insect population growth. The article contributes to the development of modern ecosystem models, which can play a crucial role for farmers and ecologists in managing insect populations and protecting the environment.

Introduction

Modeling the impact of climate change on the growth of pests and profitable forces is an important part of ecosystems. Such models may display changes in insects depending on climatic factors (eg, temperature, humidity, precipitation, and ecology.

Climate change has a significant impact on all aspects of the environment, including the growth of pests and profitable forces. High temperatures, changes in humidity, an increase in precipitation, or drought can change the living conditions of insects and have a significant impact on their populations. It is exactly insects that play an important role in this area [1-16].

Climate change, which leads to an increase in temperature, changes, and other environmental changes, including the population of insects, has a deep impact. Pest insects can cause serious damage to agriculture and food production, while the useful insects play an important role in pollination and the maintenance of the ecosystem.

Date: Date of Submission 15 April, 2025; Date of Acceptance 15 May, 2025, Communicated by Mamadsho Ilolov .

2010 *Mathematics Subject Classification.* Primary 60H10; Secondary 60H30,60H99.

Key words and phrases. mathematical modeling, harmful insects, beneficial insects, climate, differential equations, ecosystem, agriculture, temperature, humidity, radiation, analytical solution.

Mathematical Modeling enables the complex processes in the population of insects to analyze and predict climate factors. Using these models, potential changes in the populations of pests and beneficial insects, as well as effective measures to protect the environment and ensure food security, can be identified.

The report of the issue. It is required that the impact of climate change be modeled on pests and useful insects. The model should contain temperature, precipitation, soil moisture levels, and radiation levels. The goal is to analyze the balance between the impact of pests and the impact of climate change on agriculture and ecology.

In order to model this process, we will build the conceptual model. The conceptual model is the initial stage of the Mathematical Model development, which describes the research process simply and systematically. To simplify the impact of climate change on populations of pests and useful, the model should contain all important elements and ties between them. Below, the conceptual model of the issue [5-6]:

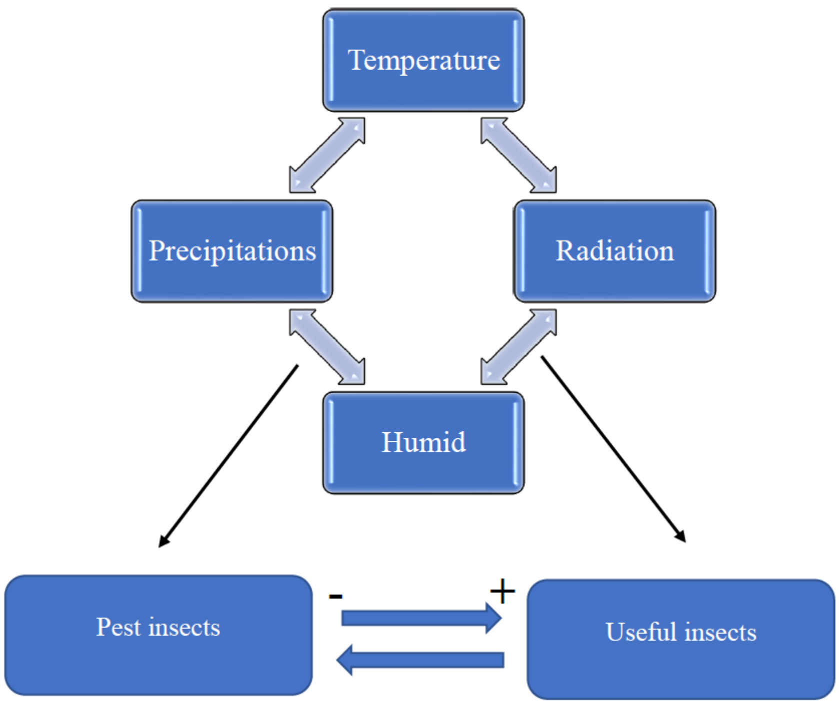

Fig. 1. Conceptual model of climate change on growth of pest and useful insects

- Model parameters:
- $N_z(t)$— the number of pests in time t;
- $N_f(t)$ — the number of useful insects in time t;
- $T(t)$ — air temperature at time t;
- $P(t)$ — the amount of precipitation at the time t;
- $W(t)$ — soil moisture;
- $R(t)$ — the surface of the solar radiation;

• T_{opt^z}, T_{opt^f} — optimal temperatures for pests and useful insects;
• $T_{max^z}, T_{min^z}, T_{max^f}, T_{min^f}$ — maximum temperature and minimum residence for each group;
• α_z, α_f — the growth rate of pest and useful insects;
• β, γ — the reduction coefficients of populations due to interacting effects.

The model consists of a system of differential equations that shows changes in populations due to external climatic conditions:

$$\begin{cases} \frac{dN_z}{dt} = \alpha_z N_z\left(t\right) \cdot \left(1 - \frac{\left|T(t) - T^z_{opt}\right|}{T^z{}_{max} - T^z{}_{min}}\right) \cdot W\left(t\right) \cdot R\left(t\right) - \beta \cdot N_z(t) \cdot N_f(t) \\ \frac{dN_f}{dt} = \alpha_f N_f\left(t\right) \cdot \left(1 - \frac{\left|T(t) - T^f_{opt}\right|}{T^f{}_{max} - T^f{}_{min}}\right) \cdot W\left(t\right) \cdot R\left(t\right) - \gamma \cdot N_f(t) \end{cases}$$

The temperature and precipitation can be modeled in the form of seasonal change:

$$T\left(t\right) = T_m + \triangle Tt, Pt = P_m + \triangle Pt,$$

Here T_m, P_m - the temperature and average of annual rainfall, $\triangle Tt$ and $\triangle Pt$ – long time and short time changes of climate.

To address the analytical solution, we analyze the model to simplify the equation and limit the model. Original equations are resolved due to complex dependence on temperatures, humidity, and interaction, often with numerous methods. But we can consider a simplified situation [7].

To simplify, let temperature $T(t), damp(t)$, and sun radiation $r(t)$. That is, they are just a middle cost to get it as constant:

$$f_z\left(T(t), W\left(t\right), R(t)\right) = k_z, f_f\left(T(t), W\left(t\right), R(t)\right) = k_f$$

These values can be taken as the average climate effect on the populations of insects.

As a result, the equation system will be simplified:

$$\begin{cases} \frac{dN_z}{dt} = \alpha_z k_z N_z\left(t\right) - \beta \cdot N_z(t) \cdot N_f(t) \\ \frac{dN_f}{dt} = \alpha_f k_f N_f\left(t\right) - \gamma \cdot N_f(t). \end{cases}$$

To find N_f consider the second equation, ie

$$\frac{dN_f}{dt} = \alpha_f k_f N_f\left(t\right) - \gamma \bullet N_f(t)$$

This equation is resolved by sorting the variables:

$$\frac{dN_f}{N_f(t)} = (\alpha_f k_f - \gamma)dt$$

$$\int \frac{dN_f}{N_f(t)} = \int (\alpha_f k_f - \gamma)dt$$

$$lnN_f\left(t\right) = \left(\alpha_f k_f - \gamma\right)t + C$$

$$N_f\left(t\right) = N_f\left(0\right)\exp\left(\alpha_f k_f - \gamma\right)t$$

It's an exponential solution to populations of beneficial insects.
If $\alpha_f k_f > \gamma$, then population is growing, otherwise.
Now to find N_z consider the first equation of the system:

$$\frac{dN_z}{dt} = \alpha_z k_z N_z\left(t\right) - \beta \cdot N_z(t) \cdot N_f(t)$$

$$\frac{dN_z}{dt} = (\alpha_z k_z - \beta \cdot N_f\left(0\right) \exp\left(\left(\alpha_f k_f - \gamma\right) t\right)) N_z$$

$$\frac{dN_z}{N_z(t)} = (\alpha_z k_z - \beta \cdot N_f\left(0\right) \exp(\left(\alpha_f k_f - \gamma\right) t)) dt$$

$$\int \frac{dN_z}{N_z(t)} = \int \alpha_z k_z dt - \int (\beta \cdot N_f\left(0\right) \exp\left(\left(\alpha_f k_f - \gamma\right) t\right)) dt$$

$$lnN_z\left(t\right) = (\alpha_z k_z t - \frac{\beta \cdot N_f\left(0\right) \exp(\left(\alpha_f k_f - \gamma\right) t)}{\alpha_f k_f - \gamma}) + C$$

$$N_z\left(t\right) = N_z\left(0\right) \exp\left(\alpha_z k_z t - \frac{\beta \cdot N_f\left(0\right) \exp(\left(\alpha_f k_f - \gamma\right) t)}{\alpha_f k_f - \gamma}\right).$$

If the values k_z and k_f - will take place in the solutions we got this kind expression:

$$N_z\left(t\right) = N_z\left(0\right) \exp\left(\alpha_z \left(1 - \frac{\left|T\left(t\right) - T^z_{opt}\right|}{T^z{}_{max} - T^z{}_{min}}\right) \cdot W\left(t\right) \cdot R\left(t\right) t - \frac{\beta \cdot N_f\left(0\right) \exp(\left(\alpha_f k_f - \gamma\right) t)}{\alpha_f k_f - \gamma}\right),$$

$$N_f\left(t\right) = N_f\left(0\right) \exp\left(\alpha_f \left(1 - \frac{\left|T\left(t\right) - T^f_{opt}\right|}{T^f{}_{max} - T^f{}_{min}}\right) \cdot W\left(t\right) \cdot R\left(t\right) - \gamma\right) t.$$

These solutions found mean that in $N_f(t)$ if $\alpha_f k_f > \gamma$, the population of profitable insects increases at the exponential rate; otherwise, the population is reduced. In a solution $N_z(t)$, the first part of the solution $exp(\alpha_z k_z t)$ the description of the growth of pest populations in conditions without the effects of profitable insects. The second part $\frac{\beta \cdot N_f(0) \exp((\alpha_f k_f - \gamma)t)}{\alpha_f k_f - \gamma}$ shows that with the effects of profitable insects are limited pest insects [7-10].

To state the stability of ecosystems, the parameters should meet the same conditions:

$$\alpha_f k_f > \gamma, \alpha_z k_z > \frac{\beta \cdot N_f\left(0\right)}{\alpha_f k_f - \gamma}$$

This means that the beneficial insects should meet climatic conditions and environmental factors to maintain sustainable growth.

Mathematical modeling of the impact of climate change to populations of pests and useful insects, developed through the system of differential equations:

1. The solution of the model indicates that the growth of insects is closely dependent on climatic factors, such as temperatures, humidity and sun radiation and sun radiation. In appropriate conditions, populations have exponential growth.

2. The equations show that temperature and other climatic factors have different impacts on the growth of pests and profitable instruments. For example, if the temperature is significantly removed from optimal temperatures, the podus growth is reduced.

3. The solutions show that the interaction between pests and useful insects (through the β and γ parameters) can affect the decline or increase in populations.

4. Potential changes in insects are predicted and allowing the possibility of taking effective measures to protect agriculture and the environment.

Thus, the model simplifies the realization of complex processes of climate effects on the ecosystem and the development of adaptation strategies and ecology.

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