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EXACT PERIODIC SOLUTION OF THE ELECTROMAGNETOELASTICITY PROBLEM FOR A FERROMAGNETIC AND SEGMENTOELECTRIC ENVIRONMENT

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Abstract. One of the main tasks in the theory of differential equations and systems is to obtain exact solutions, which lead to serious calculations. However, we do not always succeed in finding actual solutions. Our paper considers a system of nonlinear differential equations that describes electromagnetoelasticity problems for a segmentoelectric and ferromagnetic medium. Recently, different methods have been developed for solving nonlinear differential equations, one of which is the method of decomposition by elliptic Jacobi functions, which we used to obtain the solution in our paper.

Let's consider a system of partial differential equations of the form

$$\begin{cases} \dfrac{\partial \sigma_x}{\partial x} - \rho \dfrac{\partial^2 u}{\partial t^2} = f(x,t), \\ -\dfrac{\partial H}{\partial x} = \dfrac{\partial D(E)}{\partial t} + J(E) + J_{ct}, \\ \dfrac{\partial E}{\partial x} = -\dfrac{\partial B(H)}{\partial t}, \end{cases} \tag{1}$$

with the defining equations

$$\begin{cases} \sigma_x = \widetilde{E}\varepsilon_x + \widetilde{\varepsilon}E_x, \ \varepsilon_x = \dfrac{\partial u}{\partial x}, \\ D(E,\varepsilon) = \widetilde{\varepsilon}E + \varepsilon, \ \varepsilon = u, \\ B(H) = -\mu H^2 + \dfrac{\partial^2 H}{\partial t^2}, \\ f(x,t) = J(E) = J_{ct} = 0, \end{cases} \tag{2}$$

where, $\widetilde{E}, \widetilde{\varepsilon}, \mu-$ are not zero constant.

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Substituting (2) into the system of partial differential equations (1) we arrive at

$$\begin{cases} \widetilde{E}\dfrac{\partial^2 u}{\partial x^2} + \widetilde{\varepsilon}\dfrac{\partial^2 E}{\partial t^2} - \rho\dfrac{\partial^2 u}{\partial t^2} = 0, \\ -\dfrac{\partial H}{\partial x} = \widetilde{E}\dfrac{\partial E}{\partial t} + \dfrac{\partial u}{\partial t}, \\ \dfrac{\partial E}{\partial x} = -2\mu H\dfrac{\partial H}{\partial t} + \dfrac{\partial^3 H}{\partial t^3}. \end{cases} \tag{3}$$

For the problem (1)-(3) we will look for wave solutions in the form of elliptic Jacobi functions $sn\zeta, cn\zeta$ and $dn\zeta$.

For this purpose, by substituting variables of the form $\zeta = k(x - ct)$ (where k and c are constant wave numbers) for the functions

For this purpose, by substituting variables of the form $\zeta = k(x - ct)$ (where k and c constant wave numbers) for functions

$$E(x,t) = E(\zeta),\ H(x,t) = H(\zeta), u(x,t) = u(\zeta), \tag{4}$$

we obtain an ordinary system of differential equations in the following form

$$\begin{cases} \widetilde{\varepsilon}\dfrac{d^2 E}{d\zeta^2} + \overline{d}\dfrac{d^2 u}{d\zeta^2} = 0,\ (\overline{d} = \widetilde{E} - \rho c^2), \\ \dfrac{dH}{d\zeta} - c\widetilde{\varepsilon}\dfrac{dE}{d\zeta} - c\dfrac{du}{d\zeta} = 0, \\ \dfrac{dE}{d\zeta} + 2\mu c H\dfrac{dH}{d\zeta} - c^3k^2\dfrac{d^3 H}{d\zeta^3} = 0. \end{cases} \tag{5}$$

The system (5), which provides a solution to the problem (1)-(3), will be searched for in the form of a traveling wave. For this purpose, we will use the decomposition method by elliptic Jacobi functions. A similar method was used in the works [1]-[6].

Thus, we find the periodic solution (5) in the form of finite series,

$$\begin{cases} E = a_0 + a_1 sn\zeta + a_2 sn^2\zeta,\ H = b_0 + b_1 sn\zeta + b_2 sn^2\zeta, \\ u = c_0 + c_1 sn\zeta + c_2 sn^2\zeta, \end{cases} \tag{6}$$

where $a, a_1, a_2, b_0, b_1, b_2, c_0, c_1, c_2$ and c_2 as yet unknown constants.

To calculate derivatives of functions E, H and u we use formulas from elliptic function theories, i.e.

$$\frac{d(sn\zeta)}{d\zeta} = cn\zeta dn\zeta,\ \frac{d(cn\zeta)}{d\zeta} = -sn\zeta dn\zeta,\ \frac{d(dn\zeta)}{d\zeta} = -m^2 cn\zeta sn\zeta, \tag{7}$$

and identities $sn^2\zeta + cn^2\zeta = 1,\ m^2sn^2\zeta + dn^2\zeta = 1$, with a module $m(0 < m < 1)$.

Hence, using (7), and substituting (6) in the system of ordinary equations (5), we determine the constant coefficients of (6) after unsupervised transformations.

$$\begin{cases} a_0 = 0,\ a_1 = 0,\ a_2 = \frac{6ck^2m^2\overline{d}}{\mu\widetilde{\varepsilon}(\overline{d}-1)}, \\ b_0 = \frac{1}{2\mu c}[-\frac{\overline{d}}{c\widetilde{\varepsilon}(\overline{d}-1)} - 4c^3k^2(m^2+1)],\ b_1 = 0,\ b_2 = \frac{6c^2k^2m^2}{\mu} \\ c_0 = 0, /c_1 = 0,\ c_2 = -\frac{6ck^2m^2}{\mu(\overline{d}-1)}. \end{cases} \tag{8}$$

Thus, we obtain an exact periodic solution of the problem (1)-(3) by means of $sn\zeta$

$$\begin{cases} E = \frac{6ck^2m^2\overline{d}}{\mu\widetilde{\varepsilon}(\overline{d}-1)}sn^2\zeta, \\ H = -\frac{1}{2\mu c}[\frac{\overline{d}}{c\widetilde{\varepsilon}(\overline{d}-1)} + 4c^3k^2(m^2+1)] + \frac{6c^2k^2m^2}{\mu}sn^2\zeta, \\ u = -\frac{6ck^2m^2}{\mu}sn^2\zeta. \end{cases}$$

at $\mu \neq, 0/\widetilde{\varepsilon} \neq 0,\ \overline{d} \neq 1,\ c \neq 0$, or passing to the initial variations we obtain

$$\begin{cases} E(\zeta) = E(k(x-ct)) = \frac{6ck^2m^2\overline{d}}{\mu\widetilde{\varepsilon}(\overline{d}-1)}sn^2k(x-ct), \\ H(\zeta) = H(k(x-ct)) = -\frac{1}{2\mu c}[\frac{\overline{d}}{c\widetilde{\varepsilon}(\overline{d}-1)} + 4c^3k^2(m^2+1)] + \frac{6c^2k^2m^2}{\mu}sn^2k(x-ct), \\ u(\zeta) = u(k(x-ct)) = -\frac{6ck^2m^2}{\mu}sn^2k(x-ct), \end{cases} \tag{9}$$

at $\mu \neq, 0/\widetilde{\varepsilon} \neq 0,\ \overline{d} \neq 1,\ c \neq 0$.

So it's been proven,

Theorem 1. *Let all the coefficients of the system of equations (5) be different from zero except that $\mu \neq, 0/\widetilde{\varepsilon} \neq 0,\ \overline{d} \neq 1,\ c \neq 0$. Then problem (1)-(3) has an exact periodic solution of the form (9).*

Now, using the above method, we will search for the solution of the problem (1)-(3) using $cn\zeta$ Jacobi function

$$\begin{cases} E = a_0 + a_1 cn\zeta + a_2 cn^2\zeta,\ H = b_0 + b_1 cn\zeta + b_2 cn^2\zeta, \\ u = c_0 + c_1 cn\zeta + c_2 sn^2\zeta. \end{cases} \tag{10}$$

Substituting (10) using (7) into the ordinary system (5), we determine the unknown constant coefficients (10)

$$\begin{cases} a_0 = 0,\ a_1 = 0,\ a_2 = -\frac{6ck^2m^2\overline{d}}{\mu\widetilde{\varepsilon}(\overline{d}-1)}, \\ b_0 = \frac{1}{2\mu c}[-\frac{\overline{d}}{c\widetilde{\varepsilon}(\overline{d}-1)} + 4c^3k^2(2m^2-1)],\ b_1 = 0,\ b_2 = -\frac{6c^2k^2m^2}{\mu} \\ c_0 = 0,\ c_1 = 0,\ c_2 = \frac{6ck^2m^2}{\mu(\overline{d}-1)}, \end{cases} \tag{11}$$

which defines to us the following exact bounded solutions with $cn\zeta$ Jacobi functions of the form

$$\begin{cases} E = -\frac{6ck^2m^2\overline{d}}{\mu\widetilde{\varepsilon}(\overline{d}-1)}cn^2\zeta, \\ H = \frac{1}{2\mu c}[-\frac{\overline{d}}{c\widetilde{\varepsilon}(\overline{d}-1)} + 4c^3k^2(2m^2-1)] - \frac{6c^2k^2m^2}{\mu}cn^2\zeta, \\ u = \frac{6ck^2m^2}{\mu}cn^2\zeta, \end{cases}$$

on condition $\mu \neq, 0/\widetilde{\varepsilon} \neq 0,\ \overline{d} \neq 1,\ c \neq 0$.

Thus, the following theorem is proved

$$\begin{cases} E(\zeta) = E(x,t) = -\frac{6ck^2m^2\overline{d}}{\mu\widetilde{\varepsilon}(\overline{d}-1)}cn^2(k(x-ct)), \\ H(\zeta) = H(x,t) = \frac{1}{2\mu c}[-\frac{\overline{d}}{c\widetilde{\varepsilon}(\overline{d}-1)} + 4c^3k^2(2m^2+1)] - \frac{6c^2k^2m^2}{\mu}cn^2(k(x-ct)), \\ u(\zeta) = u(x,t) = \frac{6ck^2m^2}{\mu(\overline{d}-1)}cn^2(k(x-ct)), \end{cases} \tag{12}$$

on condition $\mu \neq, 0/\widetilde{\varepsilon} \neq 0,\ \overline{d} \neq 1,\ c \neq 0$.

So, it's been proven

Theorem 2. *Let all the coefficients of the system of equations (5) be different from zero except that* $\mu \neq, 0/\widetilde{\varepsilon} \neq 0,\ \overline{d} \neq 1,\ c \neq 0$. *Then problem (1)-(3) has an exact periodic solution of the form (12).*

Similarly, we can obtain an exact periodic solution of the problem with the help of $dn\zeta$ Jacobi functions in the following form

$$\begin{cases} E = -\frac{6ck^2m^2\overline{d}}{\mu\widetilde{\varepsilon}(\overline{d}-1)}dn^2(k(x-ct)), \\ H = -\frac{1}{2\mu c}[\frac{\overline{d}}{c\widetilde{\varepsilon}(\overline{d}-1)} + 4c^3k^2(m^2-2)] - \frac{6c^2k^2}{\mu}dn^2(k(x-ct)), \\ u = \frac{6ck^2}{\mu(\overline{d}-1)}dn^2(k(x-ct)), \end{cases} \tag{13}$$

on condition $\mu \neq, 0/\widetilde{\varepsilon} \neq 0,\ \overline{d} \neq 1,\ c \neq 0$.

Thus, the following theorem is proved

Theorem 3. *Let all coefficients of the system of equations* (5) *be different from zero except that* $\mu \neq, 0/\widetilde{\varepsilon} \neq 0,\ \overline{d} \neq 1,\ c \neq 0$. *Then problem (1)-(3) has an exact periodic solution of the form (13).*

Now, we consider a system of partial differential equations of the form (1) with governing equations of the form

$$\begin{cases} \sigma_x = \widetilde{E}\varepsilon_x + \widetilde{\varepsilon}E_x,\ \varepsilon_x = \dfrac{\partial u}{\partial x}, \\ D(E,\varepsilon) = \widetilde{\varepsilon}E^3 + \alpha\varepsilon,\ \varepsilon = u, \\ J(E) = \widetilde{\sigma}E_{xxx}, \\ B(H) = \mu H, \\ f(x,t) = J_{ct} = 0,\ \widetilde{E}, \widetilde{\varepsilon}, \widetilde{\sigma}, \mu, \alpha - const. \end{cases} \tag{14}$$

Using (14) from (1) we obtain

$$\begin{cases} \widetilde{E}\dfrac{\partial^2 u}{\partial x^2} + \widetilde{\varepsilon}\dfrac{\partial^2 E}{\partial x^2} - \rho\dfrac{\partial^2 u}{\partial t^2} = 0, \\ -\dfrac{\partial H}{\partial x} = 3\widetilde{\varepsilon}E^2\dfrac{\partial E}{\partial t} + \alpha\dfrac{\partial u}{\partial x} + \widetilde{\varepsilon}\dfrac{\partial^3 E}{\partial x^3}, \\ \dfrac{\partial E}{\partial x} = -\mu\dfrac{\partial H}{\partial t}. \end{cases} \tag{15}$$

Thus, in the system (15) with the help of replacement of variables for the function E, H, u types

$$E(x,t) = E(\zeta),\ H(x,t) = H(\zeta), u(x,t) = u(\zeta), \tag{16}$$

where $\zeta = k(x-ct)$, we arrive at k an ordinary system of differential equations

$$\begin{cases} \overline{d}\dfrac{d^2u}{d\zeta^2} + \overline{\varepsilon}\dfrac{d^2E}{d\zeta^2} = 0,\ (\overline{d} = \widetilde{E} - \rho c^2), \\ \dfrac{dH}{\partial\zeta} - 3c\widetilde{\varepsilon}E^2\dfrac{dE}{d\zeta} - \alpha c\dfrac{du}{d\zeta} + \widetilde{\sigma}k^2\frac{d^3E}{d\zeta^3} = 0, \\ \dfrac{dE}{d\zeta} - \mu cH\dfrac{dH}{d\zeta} = 0. \end{cases} \tag{17}$$

To solve the system (17) we also use the method of expansion by elliptic Jacobi functions. Thus, investigating by this method we will search for solutions of (17) using the elliptic Jacobi function $sn\zeta$ in the form of

$$E = a_0 + a_1 sn\zeta,\ H = b_0 + b_1 sn\zeta, u = c_0 + c_1 sn\zeta, \tag{18}$$

where $a_0, a_1, b_0, b_1, c_0, c_1$ as yet unknown constants. Hence supplying (18) in the system of ordinary equations (17), and after simple transformations we determine the unknown coefficients (18)

$$\begin{cases} a_0 = 0,\ a_1 = \pm\sqrt{\frac{2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}km, \\ b_0 = 0,\ b_1 = \pm\sqrt{\frac{2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{km}{\mu c}, \\ c_0 = 0,\ c_1 = \mp\sqrt{\frac{2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{\widetilde{\varepsilon}km}{\overline{d}}. \end{cases} \tag{19}$$

Hence, we define the exact periodic solution of the system (17) in the form

$$\begin{cases} E = \pm\sqrt{\frac{2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}kmsn\zeta, \quad H = \pm\sqrt{\frac{2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{km}{\mu c}sn\zeta, \\ u = \mp\sqrt{\frac{2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{\widetilde{\varepsilon}km}{\overline{d}}sn\zeta, \end{cases}$$

at

$$\begin{cases} 1.\widetilde{\sigma} > 0,\ c > 0,\ \widetilde{\varepsilon} > 0,\ \overline{d} \neq 0,\ \mu c \neq 0, \\ 2.\widetilde{\sigma} < 0,\ c < 0,\ \widetilde{\varepsilon} > 0,\ \overline{d} \neq 0,\ \mu c \neq 0, \\ 3.\widetilde{\sigma} < 0,\ c > 0,\ \widetilde{\varepsilon} < 0,\ \overline{d} \neq 0,\ \mu c \neq 0, \\ m^2 + 1 = \frac{\overline{d} + \alpha c^2 \mu \widetilde{\varepsilon}}{\widetilde{\sigma} k^2 \mu c \overline{d}},\ (\widetilde{\sigma} k^2 \mu c \overline{d} \neq 0) \end{cases}$$

or moving on to old changes,

$$\begin{cases} E(x,t) = \pm\sqrt{\frac{2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}kmsn(k(x-ct)), \quad H(x,t) = \pm\sqrt{\frac{2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{km}{\mu c}sn(k(x-ct)), \\ u(x,t) = \pm\sqrt{\frac{2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{\widetilde{\varepsilon}km}{\overline{d}}sn(k(x-ct)), \end{cases} \tag{20}$$

at

$$\begin{cases} 1.\widetilde{\sigma} > 0,\ c > 0,\ \widetilde{\varepsilon} > 0,\ \overline{d} \neq 0,\ \mu c \neq 0, \\ 2.\widetilde{\sigma} < 0,\ c < 0,\ \widetilde{\varepsilon} > 0,\ \overline{d} \neq 0,\ \mu c \neq 0, \\ 3.\widetilde{\sigma} < 0,\ c > 0,\ \widetilde{\varepsilon} < 0,\ \overline{d} \neq 0,\ \mu c \neq 0, \\ m^2 + 1 = \frac{\overline{d} + \alpha c^2 \mu \widetilde{\varepsilon}}{\widetilde{\sigma} k^2 \mu c \overline{d}},\ \widetilde{\sigma} k^2 \mu c \overline{d} \neq 0. \end{cases} \tag{21}$$

So, it's been proven

Theorem 4: *Let all coefficients (17) cancel from zero, and, conditions (21) are satisfied. Then the system of equations (15) has an exact periodic solution of the form (20).*

Similarly, we will search for the solution of the system of equations (17) using $cn\zeta-$ Jacobi function

$$E = a_0 + a_1 cn\zeta,\ H = b_0 + b_1 cn\zeta, u = c_0 + c_1 cn\zeta, \tag{22}$$

where too, like everywhere else $a_0, a_1, b_0, b_1, c_0, c_1-$ as yet unknown constants.

Thus, substituting (22) into the system of ordinary differential equations (17) and equating the coefficients at the same degrees $cn\zeta$ to zero, we define the coefficients of (22) in the form of

$$\begin{cases} a_0 = 0,\ a_1 = \pm\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}km, \\ b_0 = 0,\ b_1 = \pm\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{km}{\mu c}, \\ c_0 = 0,\ c_1 = \mp\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{\widetilde{\varepsilon}km}{\overline{d}}. \end{cases} \tag{23}$$

Hence, we obtain the following exact periodic solution of the system of equations (5) with respect to $cn\zeta$

$$\begin{cases} E = \pm\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}kmcn\zeta, \quad H = \pm\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{km}{\mu c}cn\zeta, \\ u = \mp\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{\widetilde{\varepsilon}km}{\overline{d}}cn\zeta, \end{cases} \tag{24}$$

on condition

$$\begin{cases} 1.\widetilde{\sigma} > 0,\ c < 0,\ \widetilde{\varepsilon} > 0,\ \overline{d} \neq 0,\ \mu c \neq 0, \\ 2.\widetilde{\sigma} < 0,\ c < 0,\ \widetilde{\varepsilon} < 0,\ \overline{d} \neq 0,\ \mu c \neq 0, \\ 3.\widetilde{\sigma} > 0,\ c > 0,\ \widetilde{\varepsilon} < 0,\ \overline{d} \neq 0,\ \mu c \neq 0, \\ 1 - 2m^2 = \frac{\overline{d} + \alpha c^2\mu\widetilde{\varepsilon}}{\widetilde{\sigma}k^2\mu c\overline{d}},\ (\widetilde{\sigma}k^2\mu c\overline{d} \neq 0). \end{cases} \tag{25}$$

Or, passing to the initial variations we obtain

$$\begin{cases} E(\zeta) = E(x,t) = \pm\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}kmcn(k(x-ct)), \\ H(\zeta) = H(x,t) = \pm\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{km}{\mu c}cn(k(x-ct)), \\ u(\zeta) = u(x,t) = \mp\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{\widetilde{\varepsilon}km}{\overline{d}}cn(k(x-ct)), \end{cases} \tag{26}$$

on condition (25).

So, it's been proven

Theorem 5: *Let all coefficients (17) be different from zero and, in addition, conditions (25) are satisfied. Then the system of equations (15) has an exact periodic solution of the form (26).*

Similarly, we define the following exact periodic solutions of the problem (1), (14) and (15) with $dn\zeta -$ delta amplitude of the Jacobi function, in the form of

$$E = a_0 + a_1 dn\zeta,\ H = b_0 + b_1 dn\zeta, u = c_0 + c_1 dn\zeta. \tag{27}$$

Following the above steps we define the coefficients (27) in the following form

$$\begin{cases} a_0 = 0,\ a_1 = \pm\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}k, \\ b_0 = 0,\ b_1 = \pm\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{k}{\mu c}, \\ c_0 = 0,\ c_1 = \mp\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{\widetilde{\varepsilon}k}{\overline{d}}. \end{cases} \tag{28}$$

Hence, we define the following exact periodic solutions of the problem (1),(14) *and* (15)

$$\begin{cases} E(x,t) = E(\zeta) = \pm\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}kdn(k(x-ct)), \\ H(x,t) = H(\zeta) = \pm\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{k}{\mu c}dn(k(x-ct)), \\ u(x,t) = u(\zeta) = \mp\sqrt{\frac{-2\widetilde{\sigma}}{c\widetilde{\varepsilon}}}\frac{\widetilde{\varepsilon}k}{\overline{d}}dn(k(x-ct)). \end{cases} \tag{29}$$

Under the following conditions

$$\begin{cases} 1.\widetilde{\sigma} > 0,\ c < 0,\ \widetilde{\varepsilon} > 0,\ \overline{d} \neq 0,\ \mu c \neq 0, \\ 2.\widetilde{\sigma} < 0,\ c > 0,\ \widetilde{\varepsilon} > 0,\ \overline{d} \neq 0,\ \mu c \neq 0, \\ 3.\widetilde{\sigma} > 0,\ c > 0,\ \widetilde{\varepsilon} < 0,\ \overline{d} \neq 0,\ \mu c \neq 0, \\ adn\ m^2 - 2 = \frac{\overline{d} + \alpha c^2 \mu \widetilde{\varepsilon}}{\widetilde{\sigma} k^2 \mu c \overline{d}},\ (\widetilde{\sigma} k^2 \mu c \overline{d} \neq 0). \end{cases} \quad (30)$$

So, the following theorem is proved

Theorem 6. *Let all the coefficients of the system of equations (17) be different from zero, and, in addition, condition (30) is satisfied. Then, problems (1), (14), and (15) have an exact periodic solution of the form (29).*

Remark. When obtaining solutions, we can also use the formula from the theory of elliptic functions, i.e.

$$sn^2\zeta + cn^2\zeta = 1, \quad dn^2\zeta = 1 - m^2 sn^2\zeta. \quad (31)$$

References

[1] M.J Ablowitz, P.A Clarkson. Solitons, nonlinear evolution equations and inverses. *Scattering transformation*, Cambridge University Press, Cambridge, 1991. (1988) 483.

[2] S.K. Liu, Z.T. Fu, S.D. Liu, Q. Zhao. Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations // J. Physics Letters A. – Moscow, 2001. – Vol. 289. – p. 69-74.

[3] N.A. Kudryashov. Analytical theory of nonlinear differential equations. - Moscow-Izhevsk: Institute of Computer Research, 2004. - 360 p. (in Russian)

[4] I. Kurbanov. Boundary value problems of electrodynamics. - Kiev: Institute of Mathematics, Academy of Sciences of the Ukrainian SSR, 1989, p. 3-23. (in Russian)

[5] I.K. Kurbanov, D.S. Safarov. Exact limited and periodic solution of the generalized B?rgers-Corteweg-de Frizas equation by constant deviating arguments// Doklady NANT.-2023.-v.67, No.7-8. (in Russian)

[6] D.S. Safarov. Exact bounded periodic solution of the generalized Korteweg-de-Vries (KdV) equation with constant deviating arguments // Reports of the NAST. – 2023. – T66, No. 5-6. – pp. 290-295.

[7] D.S. Safarov. About one generalization of the KdF-equation // Collection of scientific papers "Nonlinear boundary value problems of mathematical physics and their applications". - Kiev, 1996. - p. 240. (in Russian)

[8] D.S. Safarov. Elements of the Theory of Elliptic Functions: with Applications to Mechanics. - M.: KomKniga, 2006. - 368 p. (in Russian)

[9] A.A. Berezovskiy, I. Kurbanov. Boundary value problems of electrodynamics of conducting media. - Kiev: Institute of Mathematics, Academy of Sciences of the Ukrainian SSR, 1976, p. 37-57. (in Russian)

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